Available online at www.sciencedirect.com
ScienceDirect
Mathematics and Computers in Simulation 155 (2019) 277–289
MATHEMATICS AND COMPUTERS IN SIMULATION
www.elsevier.com/locate/matcom

Original articles

Analysis of pulse period for passive neuron in pulse coupled neural network

Rencan Nie[a,b], Jinde Cao[c,*], Dongming Zhou[b], Wenhua Qian[a,b]

[a] Jiangsu Provincial Key Laboratory of Networked Collective Intelligence, School of Automation, Southeast University, Jiangsu, Nanjing 210096, China
[b] School of Information Science and Engineering, Yunnan University, Kunming, Yunnan 650091, China
[c] School of Mathematics, Southeast University, Jiangsu, Nanjing 210096, China

Received 26 January 2018; received in revised form 16 April 2018; accepted 14 May 2018
Available online 24 May 2018

Abstract

This paper investigates the passive pulse period for the passive neuron in discrete PCNN. We first define a dynamic comparative ratio instead of the logical comparison to describe the linear difference between neural inner state and dynamic threshold. Then a nearly accurate formula about the passive pulse period is given by using the max lower limit of dynamic comparative ratios, and the rationality of which is proved based on the error analysis between estimated and real passive pulse periods. Moreover, we deduce a stable pulse period from estimated pulse period such that the neuron could nonperiodically and periodically pulse in two different time phases, successively. Further, the initial phase, from which the passive neuron can start to pulse periodically, is estimated. Some examples are performed, and the results reach the consensus with theoretical analyses.

Keywords: Pulse coupled neural network; Passive neuron; Estimated passive pulse period; Initial phase

1. Introduction

Some findings in the investigations of the visual cortices about small mammals such as cats [7,10] and guinea pig [28], have demonstrated that cortical neurons can pulse synchronously for similar stimuli. Eckhorn et al. [8] proposed a linking field model to emulate this mechanism and apply to image processing. Due to considerable nonlinearity in continuous time, Johnson et al. [18,17] modified Eckhorn's neuron model and proposed Pulse Coupled Neural Network (PCNN) for image processing. Unlike multi-layers neural networks such as recurrent neural networks [11], PCNN is a single-layer network similar to Hopfield neural networks [22] and Cohen–Grossberg neural networks [39,13]. Moreover, a neuron in PCNN is locally connected with its neighbors like in cellular neural

* Corresponding author.
E-mail addresses: rcnie@ynu.edu.cn (R.C. Nie), jdcao@seu.edu.cn (J.D. Cao), dmzhou@ynu.edu.cn (D.M. Zhou).

https://doi.org/10.1016/j.matcom.2018.05.009
0378-4754/

networks [12]. It is believed that PCNN can encode each pixel for an image into a series of pulses during iterations, and group pixels using recruitment based on similarity of intensity and space proximity. Hence PCNN has been applied well to image processing such as image segmentation [24,5,21,37,14], image fusion [19,35,9,31], and feature extraction [20,23,26,32], etc.

Especially, it is a nonlinear system [33] not only for PCNN, but also for each neuron. However, as output feedback [40], the pulses coupling among neurons in image processing is presented by using the nearly fixed local synapses, and does not consider the stochastic time delays, which exist in many stochastic systems [11,22,39,13,12,29,1,38,30]. Selecting proper parameters for PCNN neuron is the key to produce better performance in image processing, and it directly relies on the correctly understanding and the effectively analyzing the PCNN neuron how to work. [27] analyzed pulse period under an assumption that the neural inner state is fixed, it resulted in a stable period. Using strictly mathematical analysis, [15,16] proved the consensus between PCNN neuron and real biological cell. Bressloff and Coombes [2] studied on the dynamic behavior of the neuron with strong coupling, and noted that with the increase of coupling strength, the stable phase of a neuron would not be stable. Burkitt et al. [3] investigated the relation between the synchronous behavior and average stimulus for group of neurons. Note that in those of works, the analyses of PCNN were performed in continuous time and based on some assumptions. Moreover, in discrete time, Yu et al. [36] investigated the threshold of neuron on how to change the condition of fixed pulse period. [4] deduced the time phase and period when a passive neuron is pulsing, and obtained incomplete analytic formulas. The neuronic pulse period and capture characteristic for a simplified PCNN were analyzed in [25]. Suppose the decay coefficients for feedback and dynamic threshold are same, the pulse period was deduced in [6]. However, those analyses did not consider the quantization effects of integer conversion resulting from logical comparison between inner state and threshold in PCNN. Thus, those analytic results always are inaccurate.

In this paper, in discrete time, we analyzed the pulse period for passive PCNN neuron, and obtain analytic estimate for passive pulse period. The main contributions are: (1) it gives a nearly accurate estimated passive period for passive PCNN neuron by defining a comparative ratio instead of the logical comparison in PCNN; (2) it analyzes and proves the error between estimated and real passive periods; (3) it deduces the initial phase, from which the neuron starts to pulse with a stable pulse period. (4) it presents some experimental examples to verify our analysis for PCNN.

The remainder of this paper is organized as follows. In Section 2, a review of PCNN and how to vary the inner state of passive PCNN neuron are presented. Then the detailed analysis of passive pulse period is accomplished in Section 3. Some examples to verify our analysis are performed in Section 4. Eventually, the conclusion is shown in Section 5.

2. Pulse coupled neural network

A neuron in PCNN is composed of two channels. Differently, F channel not only receives coupling pulses Y from neighbors but also extrinsic stimulus S, while L channel only receives coupling pulses. Moreover, in descriptions of differential equations [30,34], the outputs of two channels all decay exponentially at each iteration.

$$F(n) = V^F Y(n-1) \otimes W + F(n-1)e^{-\alpha^F} + S \tag{1}$$

$$L(n) = V^L Y(n-1) \otimes M + L(n-1)e^{-\alpha^L} \tag{2}$$

where n is the current iteration for neuron; V^F and α^F are the magnitude and decaying coefficients for F channel, respectively, similarly V^L and α^L for L; W and M denote local synapses to neighbors for F and L respectively. Then a modulation using two-channel outputs is performed to produce inner state followed by

$$U(n) = F(n)[1 + \beta L(n)] \tag{3}$$

in which β represents the linking strength.

The pulse generator in PCNN will output a pulse when a logical comparison between the inner state and dynamic threshold is satisfied with

$$Y(n) = \begin{cases} 1, & U(n) > \theta(n) \\ 0, & otherwise \end{cases} \tag{4}$$