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Flutter and divergence instability of rectangular plates under nonconservative forces considering surface elasticity

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ABSTRACT

Nanomaterials are widely used in engineering applications. For nanostructures, surface effects usually have a significant impact on the mechanical properties of micro/nano materials and structures. This paper investigates the dynamic stability of a thin rectangular plate with surface effects under a nonconservative force. Based on the Kirchhoff plate theory incorporating surface elasticity, Hamilton's principle is employed to derive a governing partial differential equation subject to appropriate boundary conditions. A characteristic equation describing the load–frequency interaction curves is obtained. The load–frequency interaction curves are displayed graphically for various tangency coefficients. The effects of surface stress and surface mass on the frequencies and buckling loads are highlighted. Surface effects on the transition from divergence instability to flutter instability are analyzed. The surface stress and surface mass play a crucial role in affecting flutter loads of nanostructural instability. The obtained results are of benefit to safety design of micro/nano scale plates subjected to compressive loading and generalized follower force in nonconservative systems.

1. Introduction

Plates as a part of structures have been widely used in engineering applications. With the development of micro/nano-electro-mechanical system (MEMS) and human powered MEMS-based energy harvest devices, the thickness of devices may fall to micro or nano-meter order [1,2]. Especially for micro/nanometer order plate-like structures, because of the increase of specific surface area, thickness-dependent properties are notable as compared to macroscopic materials [3,4]. To account for size-dependent properties, Gurtin and Murdoch [5,6] developed a three-dimensional continuum theory with consideration of surface stress. In the context of the Gurtin–Murdoch surface elasticity theory, the surface of a solid is considered as a mathematical layer of zero thickness which has the entirely different material properties from the bulk enwrapped by the surface, and moreover there is no slip between them. Based on the extended elasticity theory incorporating surface elasticity, considerable attention has been attracted to investigate the influence of surface stress along with surface elasticity on the mechanical behavior of nanoplates or nanobeams in recent years. Lu et al. [7] proposed a thin plate theory including surface effects accounting for size-dependent static and dynamic analysis of plate-like thin film structures. Bending and bulking of Mindlin nanoplates incorporating surface energy have been studied in [8,9]. Ru [10] formulated a strain-consistent elastic plate model for various edge constraints. In addition, beam- and plate-like nanosensors can be used to identify zeptogram-scale mass [11,12]. Lachut and Sader have examined the influence of the surface stress on resonance frequencies and stiffness of thin cantilever plates. Static deflections of a cantilever beam or plate have been analyzed in [13,14]. Wang and Zhao [15] investigated the size-dependent self-buckling and bending behaviors of nanoplates with surface elasticity and surface tension. The size-dependent transverse vibration of circular and rectangular nanoplates with consideration of surface elasticity and residual stresses within the framework of the Kirchhoff theory has been respectively studied [16,17]. The effect of surface stress on the vibration and buckling of a circular and rectangular nanoplate has been analyzed by Ansari et al. [18,19]. Later, Cheng and Chen [20] extended the classical thin plate theory to include high-order surface stress and tackled resonance frequency and buckling behavior of circular and rectangular nanoplates. Yan and Jiang [21] made an analysis of the vibration and buckling behavior of a simply supported piezoelectric nanoplate with the surface effects. Wang and Wang [22] developed a continuum finite element model for the bending and vibration behaviors of nanoplates with surface effects. Recently, for a thin plate with through-thickness crack and rigid inclusion with surface effects being considered, a singular stress analysis near the crack/inclusion tip has been formulated and the surface effect on the stress intensity/singularity factors has been revealed in [23,24].

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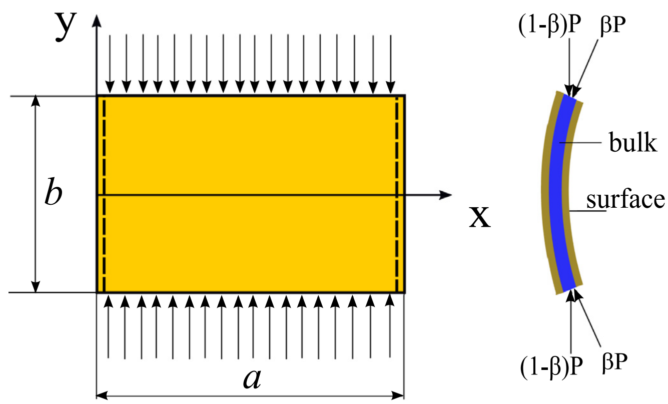

Fig. 1. Schematic of a simply supported rectangular nanoplate under uniformly distributed subtangential follower force *P*.

Although a lot of studies on free vibration and buckling of nanoplates have been reported in recent years, nearly all the related works mentioned above are focused on the condition of a conservative force. As well known, in engineering applications, most loads are nonconservative, particularly for aerodynamic and hydrodynamic loads, such as plates, beams, pipes conveying fluid and rocket subjected to nonconservative forces. When subjected to such nonconservative loads, great progress has been made for classical beams or plates [25–28]. For nanobeams subjected to nonconservative forces, the surface effect on flutter instability of a nanocantilever subjected to a generalized follower force has been studied for various boundary conditions [29–31]. For nanoplates subjected to nonconservative forces, there are little information on the work related to the surface effects on dynamic stability of nanoplates.

With the development of nanotechnology, nanomaterials and nanostructures have wide applications in engineering of a variety of fields. In this paper, we investigate dynamic instability of a micro/nano scale rectangular plate incorporating surface effects. Compared to classic plates subjected to nonconservative forces, the emphasis of this paper is placed on the influences of surface properties on the dynamic stability of plates under nonconservative force, in particular flutter and divergence instability. The surface properties include surface elasticity, surface residual stress and surface mass density. Within the framework of the Kirchhoff plate theory, we get the governing partial differential equation according to Hamilton's principle. The influences of surface effects on flutter load, divergence load and the type of instability are shown by graphs and tables in detail. The results obtained in this paper can connect scientific research with engineering application, therefore help to safety design of nanostructures exposed to a distributed force due to fluid or wind in nonconservative systems.

2. Basic equations

A schematic of a rectangular plate of length a, width b and thickness h with surface effects is shown in Fig. 1. According to the Cartesian coordinate system shown in Fig. 1, two opposite edges, $x = 0, a$, are assumed to be simply supported, and the other two opposite edges, $y = \pm b/2$, are subjected to a uniformly-distributed subtangential follower force with same magnitude (Fig. 1). When the thickness of the plate falls down to nanometer order, surface effects should be taken into account and cannot be neglected. For this reason, the theory of surface elasticity theory originally proposed by Gurtin and Murdoch [5] is invoked in the following analysis. As a result, the constitutive equations read

$$\sigma_{ij} = \lambda\varepsilon_{kk}\delta_{ij} + 2\mu\varepsilon_{ij}, \tag{1}$$

for bulk material, where λ and μ are Lame constants, δ_{ij} the Kronecker delta, ε_{ij} the strain tensor, σ_{ij} the stress tensor, $\varepsilon_{ij} = 0.5(u_{i,j} + u_{j,i})$, and u_j the displacement vector. For surface material adhere to a bulk material, the constitutive equations read [5,32]

$$\sigma^s_{\alpha\beta} = \sigma_0\delta_{\alpha\beta} + (\lambda^s + \sigma_0)\varepsilon^s_{\gamma\gamma}\delta_{\alpha\beta} + 2(\mu^s - \sigma_0)\varepsilon^s_{\alpha\beta} + \sigma_0 u^s_{\alpha,\beta}, \tag{2}$$

$$\sigma^s_{\alpha z} = \sigma_0 u^s_{z,\alpha}, \tag{3}$$

where σ_0 is the surface residual stress, a quantity with the superscript s represents the one for the surface material. For example, λ^s and μ^s are the surface Lame constants, which are related to surface Young's modulus E^s and surface Poisson's ratio ν^s by

$$\lambda^s = \frac{E^s\nu}{(1+\nu^s)(1-2\nu^s)}, \quad \mu^s = \frac{E^s}{2(1+\nu^s)}. \tag{4}$$

In the following analysis, due to little change of Poisson's ratio, it is assumed that $\nu = \nu^s$. In the above, Latin subscripts *i, j, k* take values from 1 to 3, and Greek subscripts α, β, γ range from 1 to 2. A comma in subscript denotes differentiation with respect to the coordinate variable following. A convention that repeated indices imply summation has been used.

For a thin rectangular plate, within the framework of the Kirchhoff plate theory, the displacement components of the plate along the x-, y- and z-axes can be expressed as

$$u_x = u_0 - z\frac{\partial w}{\partial x}, \quad u_y = v_0 - z\frac{\partial w}{\partial y}, \quad u_z = w(x, y) \tag{5}$$

where the plate's deflection $w(x, y)$ is dependent on spatial variables x and y, and independent of z, and $u_0(x, y)$, $v_0(x, y)$ are the displacement components along the x, y directions of the midplane, respectively, which arise from the membrane force in the xoy-plane. Since the influence of axial displacements $u_0(x, y)$, $v_0(x, y)$ at the midplane is very small, they could be reasonably neglected in the following analysis. Therefore, the strain components in Cartesian coordinates could be derived from Eq. (5) as follows:

$$\varepsilon_{xx} = -z\frac{\partial^2 w}{\partial x^2}, \quad \varepsilon_{yy} = -z\frac{\partial^2 w}{\partial y^2}, \quad \gamma_{xy} = -2z\frac{\partial^2 w}{\partial x\partial y} \tag{6}$$

Furthermore, the stress components in the plate can be written below [33]:

$$\sigma_{xx} = \frac{E}{1-\nu^2}(\varepsilon_{xx} + \nu\varepsilon_{yy}) = -\frac{Ez}{1-\nu^2}\left(\frac{\partial^2 w}{\partial x^2} + \nu\frac{\partial^2 w}{\partial y^2}\right), \tag{7}$$

$$\sigma_{yy} = \frac{E}{1-\nu^2}(\varepsilon_{yy} + \nu\varepsilon_{xx}) = -\frac{Ez}{1-\nu^2}\left(\frac{\partial^2 w}{\partial y^2} + \nu\frac{\partial^2 w}{\partial x^2}\right), \tag{8}$$

$$\sigma_{xy} = \frac{E}{2(1+\nu)}\varepsilon_{xy} = -\frac{Ez}{1+\nu}\frac{\partial^2 w}{\partial x\partial y}, \tag{9}$$

for bulk material and

$$\sigma^{s\pm}_{xx} = \sigma_0 \mp \frac{h}{2}\left[(2\mu^s + \lambda^s)\frac{\partial^2 w}{\partial x^2} + (\lambda^s + \sigma_0)\frac{\partial^2 w}{\partial y^2}\right] \tag{10}$$

$$\sigma^{s\pm}_{yy} = \sigma_0 \mp \frac{h}{2}\left[(2\mu^s + \lambda^s)\frac{\partial^2 w}{\partial y^2} + (\lambda^s + \sigma_0)\frac{\partial^2 w}{\partial x^2}\right] \tag{11}$$

$$\sigma^{s\pm}_{xy} = \mp\frac{h(2\mu^s - \sigma_0)}{2}\frac{\partial^2 w}{\partial x\partial y} \tag{12}$$

for surface material, where signs + and – denote the upper and lower surfaces of a plate, respectively. In the above E and ν stand for Young's modulus and Poisson's ratio for bulk material, respectively.

For static and dynamic analysis of plates, bending moments play a crucial role, which arise from bulk and surface parts. In other words, the bending moments can be calculated as follows:

$$M_{\alpha\beta} = \int_{-h/2}^{h/2} z\sigma_{\alpha\beta}dz + \frac{h}{2}(\sigma^+_{\alpha\beta} - \sigma^-_{\alpha\beta}). \tag{13}$$