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On the accuracy and optimization application of an axisymmetric simplified model for underwater sound absorption of anechoic coatings

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ABSTRACT

The finite element method (FEM) is commonly used to analyze the underwater acoustic performances of rubbery coatings due to its flexibility to model scatterers of complex geometric dimensions. However, its calculation efficiency is always denounced as a drawback for its application on optimization design where a large number of repeated calculations should be fulfilled. In this paper, the accuracy of an axisymmetric simplified model, to model the unit cell as circular cross section, is systematically discussed by respectively comparing the absorption coefficients with those calculated by the square and hexagonal models under different punching rates of the cavities. Results show that the axisymmetric model can better describe the sound absorption of the coating with a hexagonal arrangement; however, it may cause comparative large deviations to describe the results of the coating with a square arrangement under not only large punching rates but also small punching rates. This phenomenon can be related to the different boundary differences between two arrangements of cavities and can be relieved when the rubber material possesses a large loss factor. Then, a shape optimization of the embedded cavity using the axisymmetric simplified model and a genetic algorithm has been done to achieve optimal sound absorption in 1–20 kHz. Compared with the coating with isochoric cylindrical cavities, the optimized coating can achieve better sound absorption in low frequencies due to the larger pore-aperture radius of the optimized cavity than that of the isochoric cylindrical cavity. The frequency-dependent parameters of the rubber material play an important role for the broadband high absorption of the optimized coating.

1. Introduction

Rubbery coatings are commonly used as attachments on underwater structures to reduce reflection of incoming sound waves [1]. To achieve a good anechoic property, the rubbery coatings should possess good sound absorption ability. Due to the fact that the compressional-wave dissipation in rubbery material is much less efficient than that of shear wave, rubber layers are usually embedded with various scatterers, such as air-filled cavities [1–14], microspheres [15–18], or locally resonant phononic crystals [19], to induce scattering and wave mode conversion, and thus to enhance the sound energy dissipation.

Cavity-type scatterers were firstly introduced into the rubber matrix to constitute the so-called Alberich anechoic coating in the Second World War [1], and have received much attention over the past few decades. Early modeling studies for rubbery coatings with cavities were mainly based on homogenization and effective medium properties [20–22] and simplified analytical models, such as the lumped system approximation [1], one-dimensional [3] and two-dimensional [4] waveguide models. In recent years, a semi-analytical method, the layer-multiple scattering method, borrowed from Phononic Crystals is extended to study the echo reduction by rubbery coatings with spherical or super-ellipsoidal cavities [7–9], microspheres [17,18] and locally resonant scatterers [10,19,23]. However, these methods require either lots of specific algebraic developments or special simplifying hypotheses, which restrict their application to a small number of given geometries.

Apart from the analytical or semi-analytical method mentioned above, the finite element method (FEM) has also been used to modeling the acoustic properties of rubbery coatings. An important advantage is the flexibility to model the scatterers with complex geometries. Periodicity can be utilized to restrict the computer intensive finite element (FE) modeling to an individual unit cell. Hladky-Hennion et al. firstly introduced FEM into the investigation of scattering of plane waves from water immersed singly/double periodic structures, such as compliant tube gratings [24] and Alberich anechoic coatings [25], by combining FE discretization of a single cell with Bloch-Floquet theory. Then, Easwaran and

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Munjal [26] presented another three-dimensional (3D) FEM to analyze the reflection characteristics of resonant sound absorbers. The difference is that the modal summation technique in the fluid domain in Ref. [25] is replaced by the specified impedance boundary conditions. From then on, this method was commonly used for the investigation of rubbery coatings with different scatterers [11–14,23,27–37].

The design problem of anechoic coatings can be formulated as a nonlinear optimization problem [7,9]. Hence, global optimization techniques from inverse theory can be applied. Coating optimization has previously been attempted using a simplex procedure with a broadband objective function based on mechanical resonator formulas [38] and using a genetic algorithm or differential evolution algorithm combined with layer-multiple-scattering method [7,9,10,29]. The modeling flexibility of FEM is useful for applications to anechoic coatings, however, its calculation efficiency is always denounced as a drawback for its application on optimization design where a large number of repeated calculations should be fulfilled.

To overcome this problem, much research efforts were focused on the approaches to improve its calculation efficiency, such as choosing a proper mesh precision and utilizing the structural symmetry. The mesh precision is known as an important factor to balance the calculation accuracy with efficiency. It has been pointed out that the $\lambda/4$ criterion for mesh precision is sufficient for finite elements relying upon a quadratic interpolation but with respect to the smallest wave velocity [25]. This velocity is classically the transverse wave velocity, but can be the flexural wave velocity in the cavity cover layers if these layers are sufficiently thin. Then, the structural symmetry was used to simplify the modeling domain into only one-quarter [26] or even one-eighth [31] of the unit cell with normal incidence and thus to drastically reduce the CPU time and the core memory requirements of the computer.

Based on these efforts, several FEM-based optimization works on anechoic coatings were carried out. Cai et al. [27] optimized the dimensions of cylindrical cavities using a zero-order method combined with FEM for a single-frequency objective function. Yang et al. [32] used a genetic algorithm combined with FEM to design a two-dimensional anechoic layer with scatterers of mixed size at 2–10 kHz for a normal incidence. Li et al. [36] proposed a topology optimization method based on FEM by modeling one-quarter of the unit cell to determine the optimal material layout of cavities to achieve maximized sound absorption. Even so, the calculation efficiency of FEM is still an important factor that restricts its application in optimization design of anechoic coatings.

In recent years, an approximate simplification to model the unit cell as circular cross section by ignoring the effect of geometrical boundary of the unit cell is proposed [5,6,39,40]. Based on this simplification, the 3D unit cell can be further simplified into a two-dimensional (2D) axisymmetric model when the internal structure is axisymmetric. The validation of this method has been claimed and supported by the results from the commercial FEM software such as *ANSYS* [41] and *COMSOL Multiphysics* [42] and other FEM code [39]. It has been claimed that this method can get both good accuracy and calculation efficiency. However, it should be noted that the validation of this method is only verified with a specific structure. To the best of the author's knowledge, there is no paper to discuss the dependence of its accuracy on different structures. Its accuracy is still questionable in the cases with various inner structures. Thus, the purpose of this paper is to further check the accuracy of the axisymmetric simplified model. More specifically, it's to investigate the relation between the accuracy of the axisymmetric simplified model with the arrangement styles and punching rates of scatterers. Then, a shape optimization of embedded cavities of anechoic coatings is carried out to achieve optimal sound absorption in 1–20 kHz by using this axisymmetric simplified model and a genetic algorithm.

2. Finite element method and axisymmetric FE model

2.1. Model description and FEM

Fig. 1(a) shows the schematic description of the analysis model. The anechoic coating is made up of rubbery materials embedded with air cavities and water-loaded at one side. The rubbery coating is bonded to a steel slab followed by semi-infinite air. A harmonic plane wave is incident from the semi-infinite water domain and perpendicular to the coating surface. As shown in Fig. 1(b) and (c), the embedded cavities are in a square or hexagonal arrangement along the *xoy* plane, and the axes of the cavities are parallel to the *z* direction. In this paper, the finite element software COMSOL Multiphysics® (v5.1) [43] is used to model this problem. Due to the periodicity of the inner cavities, only one unit cell of the periodic structure has to be modeled by using the periodic boundary conditions. Fig. 1(b) and (c) show the representative unit cells for the square and hexagonal arrangement of air cavities. The structural symmetry can be further utilized to simplify the modeling domain into only one-eighth square cell and one-quarter hexagonal cell, respectively represented by the red shadow areas shown in Fig. 1(b) and (c) at a normal incidence. The discretized form of the governing equation for the coupled structure-acoustic problem can be written in matrix form as follows [11,12]:

$$\begin{bmatrix} \mathbf{M_s} & 0 \\ \rho_f \mathbf{R} & \mathbf{M_f} \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{u}}_\mathbf{e} \\ \ddot{\mathbf{p}}_\mathbf{e} \end{Bmatrix} + \begin{bmatrix} \mathbf{C_s} & 0 \\ 0 & \mathbf{C_f} \end{bmatrix} \begin{Bmatrix} \dot{\mathbf{u}}_\mathbf{e} \\ \dot{\mathbf{p}}_\mathbf{e} \end{Bmatrix} + \begin{bmatrix} \mathbf{K_s} & -\mathbf{R} \\ 0 & \mathbf{K_f} \end{bmatrix} \begin{Bmatrix} \mathbf{u_e} \\ \mathbf{p_e} \end{Bmatrix} = \begin{Bmatrix} \mathbf{F_s} \\ \mathbf{F_f} \end{Bmatrix}, \tag{1}$$

where $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ are the mass, damping, and stiffness matrices, respectively. The subscripts s and f denote the structural and the fluid domains, respectively. $\mathbf{u_e}$ is the nodal displacement vector in the structural domain and $\mathbf{p_e}$ is the nodal pressure vector in the acoustic domain. $\mathbf{F_s}$ and $\mathbf{F_f}$ are the nodal structural force and the nodal acoustic pressure vectors, respectively. $\mathbf{R}$ is the coupling matrix representing the coupling conditions on the interface between the acoustic fluid and the structure. ρ_f is the density of the fluid medium. The rubber matrix is modeled as a solid domain, the water and air domains are fluid media and the inner cavities are evacuated. Only finite domains of water and air are modeled with the perfectly matched layers added at both the top of water domain and the bottom of air domain to mimic anechoic termination of outgoing waves [12,30].

Combining the boundary conditions to solve the Eq. (1), one can obtain the displacement $\mathbf{u_e}$ for the structure and the pressure $\mathbf{p_e}$ in the fluid, and then the reflection R and transmission coefficients T can be obtained [25,28,35]. The sound absorption coefficient α can then be calculated from

$$\alpha = 1 - R^2 - T^2. \tag{2}$$

2.2. Simplified 2D axisymmetric FE model

Fig. 2 shows the schematic diagram of the simplified axisymmetric FE model. The square or hexagonal external boundaries are approximately replaced by the circular external boundary and thus the square and/or hexagonal cells are approximated into circular cell. During the approximation, the cavity dimensions and the punching rate are kept the same. The circular cell can further be simplified into a 2D axisymmetric model when the internal cavity is axisymmetric, such as cylindrical or conical. To mimic the periodic boundary at a normal incidence, the boundary condition