Engineering Structures 177 (2018) 770–784

Contents lists available at ScienceDirect

Engineering Structures

journal homepage: www.elsevier.com/locate/engstruct

Higher order beam theory for linear local buckling analysis

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ARTICLE INFO

Keywords:
Distortion
Warping
In-plane deformation
Out-of-plane deformation
Linear buckling
Local buckling
Axial mode
Flexural mode
Torsional mode
Beam

ABSTRACT

In this paper, a higher order beam theory is employed for linear local buckling analysis of beams of homogeneous cross-section, taking into account warping and distortional phenomena due to axial, shear, flexural, and torsional behavior. The beam is subjected to arbitrary concentrated or distributed loading, while its edges are restrained by the most general linear boundary conditions. The analysis consists of two stages. In the first stage, where the Boundary Element Method is employed, a cross-sectional analysis is performed based on the so-called sequential equilibrium scheme establishing the possible in-plane (distortion) and out-of-plane (warping) deformation patterns of the cross-section. In the second stage, where the Finite Element Method is employed, the extracted deformation patterns are included in the buckling analysis multiplied by respective independent parameters expressing their contribution to the beam deformation. The four rigid body displacements of the cross-section together with the aforementioned independent parameters constitute the degrees of freedom of the beam. The finite element equations are formulated with respect to the displacements and the independent warping and distortional parameters. The buckling load is calculated and is compared with beam and 3d solid finite elements analysis results in order to validate the method and demonstrate its efficiency and accuracy.

1. Introduction

In most cases, in the analysis of beam-like structures, Euler – Bernoulli beam theory assumptions are adopted, while in the case of non-negligible shear deformation effect these assumptions are relaxed by using Timoshenko beam theory. However, both theories maintain the assumption that cross-section behaves as a rigid body. In order to take into account shear lag effects in the context of a beam theory, the inclusion of non-uniform warping is necessary, relaxing the assumption of plane cross-section. The shear flow associated with non-uniform warping leads also to in-plane deformation of the cross-section, relaxing the assumption that the cross-section shape does not change after deformation. For this purpose, the so-called higher order beam theories have been developed taking into account shear lag [1,2] and distortional (in-plane deformation) effects [3,4]. Higher order beam theories are of increased interest due to their important advantages over approaches such as 3-D solid or shell solutions as they:

(a) require less modelling time,
(b) permit isolation of structural phenomena and results interpretation (rotations, warping parameters, stress resultants etc. are also evaluated in addition to displacements and stress components),
(c) facilitate modelling of supports and application of external loading,
(d) require significantly less number of degrees of freedom (dofs) reducing computational time, and
(e) facilitate parametric analyses without the construction of multiple models.

Elastic stability of beams is one of the most important criteria in the design of structures. Chen et al. [5] were the first that included a simple analytical model in their beam formulation to account for the effects of local buckling of circular cross-section. Since then, numerous research efforts have been published concerning buckling including shear lag and distortional effects in a beam theory. Some researchers, have studied local buckling of beams employing Generalized Beam Theory (GBT), i.e., “a thin-walled prismatic bar theory that includes cross-section in-plane and out-of-plane(warping) deformation through the consideration of so-called cross-section deformation modes” [6]. Davies et al. used GBT to investigate the buckling of cold-formed steel (open-section) profiles [7], while Camotim et al. studied local buckling of beams regarding steel and aluminum columns [8], thin-walled regular polygon tubes, angle, T-sections and cruciform thin-walled members [9–11], cold formed steel purlins [12], steel-concrete composite beams [6] employing GBT. Other researchers, studied buckling problems of

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https://doi.org/10.1016/j.engstruct.2018.08.069
Received 18 April 2018; Received in revised form 2 August 2018; Accepted 20 August 2018

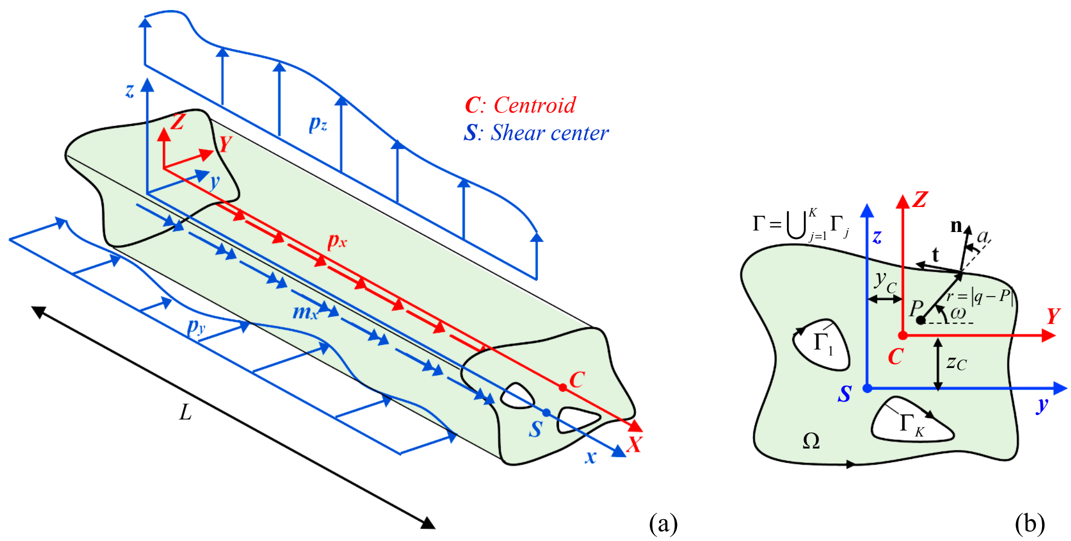

Fig. 1. Prismatic beam under loading (a) with a homogeneous cross-section of arbitrary shape occupying the two dimensional region Ω (b).

beams employing finite strip method [13–15] that relies on plate theory assumptions which are applied to each wall of the cross-section. Karamanos et al. [16–19] formulated a model that accounts for global behavior and for local buckling as well. The global (beam-type) response is described through Lagrange polynomials and the cross-sectional ovalization/warping in terms of trigonometric functions. This formulation has been quite successful in simulating local buckling in circular hollow section members [18,19]. Local buckling has also been examined for several cases of thin-walled sections [e.g., 20–23]. All the aforementioned researches deal with the problem of local buckling, however, they employ assumptions of thin tube theory and, in some cases, their application is limited by the cross-section shape.

In this paper, a higher order beam theory is employed for linear local buckling analysis [24] of beams of homogeneous cross-section, taking into account warping and distortional phenomena due to axial, shear, flexural, and torsional behavior [25,26]. The rest of the paper, except the introduction, consists of three main parts. In the first one, Section 2: "Statement of the Problem", the displacement field of the arbitrarily shaped, homogeneous beam is defined. It is employed for the derivation of the expressions of stresses and strains which along with the external actions are introduced in the principle of virtual work. In order to separate the analysis in two stages, i.e., a cross sectional analysis and a longitudinal analysis, which is the concept of a beam theory, both stresses and strains are written as a product of two matrices. The first one contains expressions which are functions of the cross-sectional coordinates, the warping functions and the components of distortional functions while the second one contains expressions which are functions of the components of the displacement vector and their derivatives. As far as the evaluation of warping and distortional functions is concerned, the so-called sequential equilibrium scheme [26] is employed establishing the possible in-plane (distortion) and out-of-plane (warping) deformation patterns of the cross-section by means of the Boundary Element Method (BEM). The cross-sectional analysis is based on [25] for axial modes and on [26] for flexural and torsional modes. In the next section, Section 3 "Numerical Solution" the Finite Element Method (FEM) is employed discretizing the beam and formulating the geometric stiffness matrix and the buckling criterion by exploiting the principal of virtual work of Section 2 and utilizing the boundary conditions. In the final main part of the paper, Section 4 "Numerical Examples" representative examples are presented where buckling load is calculated and is compared with beam and 3d solid finite elements analysis results in order to validate the proposed method and demonstrate its efficiency and accuracy. Finally, the conclusions of the present paper are summarized in Section "Conclusions", while the expressions of the matrices which form strains and stresses are provided in "Appendix A".

The essential features and novel aspects of the proposed formulation, compared with previous ones, are summarized as follows.

(i) For the first time in the literature, linear (global and local) buckling analysis is conducted based on a very general beam theory including axial, shear, flexural, and torsional warping and distortional effects, following the sequential equilibrium scheme and employing BEM.
(ii) The cross-section can be thin- or thick-walled. The formulation does not stand on the assumption of thin-walled structure.
(iii) It performs linear (global and local) buckling analysis based on a higher-order beam theory that is of increased interest due to its important advantages over refined approaches such as 3-D solid or shell solutions.
(iv) The influence of Poisson's ratio is taken into account in the linear local buckling analysis of beams.
(v) The beam is supported by the most general linear boundary conditions including elastic support or restraint.

2. Statement of the problem

2.1. Displacement components

Let us consider a prismatic beam of length L (Fig. 1a), of an arbitrarily shaped cross-section of area A (Fig. 1b). The cross-section consists of a homogeneous and isotropic material, with modulus of elasticity E and Poisson's ratio ν, occupying the two-dimensional multiply connected region Ω of the y, z plane (Fig. 1b) bounded by the Γ_j ($j = 1, 2, ..., K$) boundary curves. These curves are piecewise smooth, i.e., they may have a finite number of corners. In Fig. 1, CYZ is the principal bending coordinate system through the cross-section centroid C, while y_c, z_c are its coordinates with respect to Syz principal shear system of axes through the cross-section shear center S. Finally, it holds that $Y = y - y_c$ and $Z = z - z_c$.

The beam can be supported by the most general linear boundary conditions and is subjected to the combined action of the arbitrarily distributed or concentrated axial loading $p_x(X)$ along X direction, transverse loading $p_y(x)$ and $p_z(x)$ along the y, z directions, respectively, twisting moment $m_x(x)$ along x direction, bending moments $m_Y^P(x)$, $m_Z^P(x)$ along Y, Z directions, respectively, as well as bending $m_{\varphi_Y^S}(x)$, $m_{\varphi_Z^S}(x)$ and primary and secondary torsional $m_{\varphi_x^P}(x)$, $m_{\varphi_x^S}(x)$ and axial $m_{\varphi_u^P}(x)$, $m_{\varphi_u^S}(x)$ warping moments, and higher moments