J. Math. Anal. Appl. ••• (••••) •••–•••

Contents lists available at ScienceDirect

Journal of Mathematical Analysis and Applications

www.elsevier.com/locate/jmaa

The number and stability of limit cycles for planar piecewise linear systems of node–saddle type ☆

Jiafu Wang [a,*], Xiaoyan Chen [b], Lihong Huang [a]

[a] *School of Mathematics and Statistics, Changsha University of Science and Technology, Changsha, Hunan 410114, China*
[b] *Institute of Mathematics and Physics, Central South University of Forestry and Technology, Changsha, Hunan 410004, China*

A R T I C L E   I N F O

Article history:
Received 20 February 2017
Available online xxxx
Submitted by R. Popovych

Keywords:
Limit cycle
Poincaré–Bendixon theorem
Stability
Piecewise linear differential systems

A B S T R A C T

The objective of this paper is to study the number and stability of limit cycles for planar piecewise linear (PWL) systems of node–saddle type with two linear regions. Firstly, we give a thorough analysis of limit cycles for Liénard PWL systems of this type, proving one is the maximum number of limit cycles and obtaining necessary and sufficient conditions for the existence and stability of a unique limit cycle. These conditions can be easily verified directly according to the parameters in the systems, and play an important role in giving birth to two limit cycles for general PWL systems. In this step, the tool of a Bendixon-like theorem is successfully employed to derive the existence of a limit cycle. Secondly, making use of the results gained in the first step, we obtain parameter regions where the general PWL systems have at least one, at least two and no limit cycles respectively. In addition for the general PWL systems, some sufficient conditions are presented for the existence and stability of a unique one and exactly two limit cycles respectively. Finally, some numerical examples are given to illustrate the results and especially to show the existence and stability of two nested limit cycles.

© 2018 Published by Elsevier Inc.

1. Introduction

Piecewise smooth (PWS) or Filippov systems are widely used in a natural way to model many real processes and phenomena [1,5], such as idealized models for cell activity [3] and discontinuous control in the Buck electronic converter [6,12]. As is well known, piecewise linear (PWL) systems are the simplest possible configuration in PWS or Filippov systems. A natural question is: how many limit cycles are there in PWL systems? This problem is very related to the Hilbert's 16th problem [13].

☆ This work is supported by the National Natural Science Foundation of China (Grant Nos. 11771059, 11301551), the Natural Science Foundation of Hunan Province, China (Grant No. 2017JJ3525) and Hunan Provincial Key Laboratory of Mathematical Modeling and Analysis in Engineering, China (Grant No. 2017TP1017).

* Corresponding author.
E-mail address: jfwangmath@163.com (J. Wang).

https://doi.org/10.1016/j.jmaa.2018.09.024
0022-247X/© 2018 Published by Elsevier Inc.

Please cite this article in press as: J. Wang et al., The number and stability of limit cycles for planar piecewise linear systems of node–saddle type, J. Math. Anal. Appl. (2018), https://doi.org/10.1016/j.jmaa.2018.09.024

Table 1
The maximum number of known limit cycles of the general system (1.1) before this paper. The symbol "–" indicates that those cases appear repeated in the table.

Type	Focus	Node	Saddle
Focus	3	3	3
Node	–	2	?
Saddle	–	–	2

Recently, crossing limit cycles [6] (in the sequel, unless otherwise noted, limit cycle means crossing limit cycle) have been studied for PWL systems with two zones separated by a straight line [2,4,6–11,15–19]. By means of bifurcation techniques, Han and Zhang in [8] proved that two limit cycles can appear in a class of planar PWL systems and conjectured that the maximum number of limit cycles should be at most two for general planar PWL systems. However later on, Huan and Yang in [9] found that three limit cycles could exist by a numerical example. To cope with a sufficiently broad class of systems, the work [6] provided a Liénard-like canonical form for planar PWL systems with two zones separated by a straight line as follows

$$\dot{\mathbf{x}} = \begin{cases} \begin{pmatrix} T^+ & -1 \\ D^+ & 0 \end{pmatrix} \mathbf{x} - \begin{pmatrix} -b \\ a^+ \end{pmatrix}, \; x > 0, \\ \begin{pmatrix} T^- & -1 \\ D^- & 0 \end{pmatrix} \mathbf{x} - \begin{pmatrix} 0 \\ a^- \end{pmatrix}, \; x < 0, \end{cases} \tag{1.1}$$

where $\mathbf{x} = (x, y)^{\mathrm{T}} \in \mathbb{R}^2$. A focus, a node or a saddle of a linear differential subsystem defined in a half-plane is *real* when it belongs to the closure of the half-plane where the corresponding linear differential subsystem is defined, and it is called *virtual* otherwise. For the system (1.1) being of focus–focus type, the authors [6] showed the existence of two limit cycles where both the foci of each subsystem are virtual. Whereas, if one subsystem has a real focus, they [7] also gave a mechanism to generate three limit cycles for the system (1.1). When the system (1.1) is of saddle–saddle type, Huan and Yang [10] obtained sufficient conditions for the existence of at least two limit cycles. Furthermore, Huan and Yang [11] also investigated the existence and number of limit cycles for the system (1.1) of node–node type. In contrast to the works [6,10,11], Li [14] paid attention to the system (1.1) of saddle–focus type, showing that three limit cycles can appear in the system (1.1) with one subsystem having a real saddle point and the other a virtual focus. For the system (1.1) being of node–saddle type, to the best of our knowledge, the problem of limit cycles is still unclear. In summary, up to now for the general system (1.1), the maximum number of known limit cycles are given in Table 1. Some specific examples given in [17] evidenced the possibility of the existence of two limit cycles for the system (1.1) of node–saddle type, but the parameter region with two limit cycles remains unknown. Moreover, parameter regions where there is one or no limit cycle, seem to be unclear as well. Another interesting question is the maximum number of limit cycles for the system (1.1) in the node–saddle case. As stated before, a general mechanism [7] is presented to generate three limit cycles for the system (1.1) in the focus–saddle, focus–node and focus–focus cases. Then, in a similar way, can this mechanism generate three limit cycles in the node–saddle case?

Based on these motivations, the objective of this paper is to present the parameter regions where there is one, two and no limit cycles respectively, and further the stability of the limit cycles. As a result, we find the mechanism given in [7] could not generate three limit cycles for the system (1.1) in the node–saddle case. In order to achieve our goal, at first we should investigate the uniqueness and stability of limit cycles for the corresponding Liénard PWL system, that is, the system (1.1) with $b = 0$. This investigation can help us to give birth to two limit cycles for the general system (1.1). To be more specifically, the main novelty of this paper is to define and study a new displacement function (defined by (4.3) in Section 4), whose special

Please cite this article in press as: J. Wang et al., The number and stability of limit cycles for planar piecewise linear systems of node–saddle type, J. Math. Anal. Appl. (2018), https://doi.org/10.1016/j.jmaa.2018.09.024

Download English Version:

https://daneshyari.com/en/article/11021734

Download Persian Version:

https://daneshyari.com/article/11021734

Daneshyari.com