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Minireview

Anomalous transports in a space–time inseparable system

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A B S T R A C T

We investigate transport of an inertial particle in a spatially symmetric periodic potential, such that phase of the potential is controlled by two time periodic modulations. Our numerical results indicate that the phase modulation can induce anomalous transport of the system, while any current does not appear for only one time periodic modulation. In the absence of external bias, direction of mean velocity of the particle changes several times with amplitude and frequency of one of the modulations, i.e., multiple current reversals (CRs). The multiple CRs results from temporal symmetry breaking of the system. In the presence of external bias, there exists phenomenon of absolute negative mobility (ANM) in the system, while action of noise weakens the ANM. Intrinsic physical mechanisms and conditions responsible for occurrence of the anomalous transport are also discussed in detail.

Contents

1. Introduction

In recent years, much attention has been paid to transport behaviors of particles for both classical and quantum systems [1–8], owing to their relevance in a wide range of fields, including, biology [9–11], nanotechnology [12,13], chemistry [14], cold atoms [15–17], and Bose–Einstein condensates [18–22]. Motivated by fundamental interest and prospect of controlling particle motions in microdevices, physical mechanisms involved in transport of particles and conditions under which it can occur have been the subject of a large number of investigations over the years.

In a system with a periodic potential, symmetry breaking is a necessary ingredient for inducing abnormal dynamical behaviors of particles. It is generally thought that such symmetry breaking arises mainly in spatially periodic potential,

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temporal periodic signals, or under the influence of external constant forces or noise. Superposition of spatial periodic potentials with different periods leads to spatial ratchet [23–28], and correlation between two kinds of noises [29] can also result in symmetry breaking of system. Temporal ratchet mainly results from mixing of time-dependent periodic signals with different frequencies [30,31]. In a spatially symmetric periodic system, combination of two additive signals with different frequencies acts as a rocking force, which breaks relevant temporal symmetries and creates a current [32,33]. For a gating ratchet potential, in which one of its signals is multiplicative and used to modulate the spatial potential, directed transport is generated with lowering of potential barriers synchronized with the motion produced by an additive force [34,35]. In **addition**, there are other factors leading to temporal ratchet, such as phase modulation to be discussed in this paper. Experimentally, such system can be designed and the phase modulation has been found to cause the Rabi oscillations observed in an optical potential system [36]. In what follows, it will be seen that as phase of a spatially symmetric periodic potential is controlled by two cosine modulations, a net current can emerge even in the absence of external periodic drive force and noise, which has not yet been reported in the present study.

One of the most important anomalous transport phenomena belongs to absolute negative mobility (ANM), i.e., current of system decreases with increment of external constant force, which has been investigated both theoretically and experimentally [37–42]. In general, external periodic signal seems to be an indispensable ingredient for occurrence of ANM in many systems. For a vibrational motor, ANM was observed and an additional time-periodic signal filled the role usually played by noise in a Brownian motor [43]. However, in the gating ratchet potential, the additive periodic signal cooperates with the multiplicative periodic signal to make a particle accelerate along the opposite direction of the external constant force [44,45]. In spatially ratchet/symmetric periodic systems [46,47], an external periodic signal with appropriate amplitude and frequency is also required for occurrence of the ANM. It deserves attention that external periodic signal plays an important role in inducing anomalous transport. In fact, as phase of the symmetric periodic potential is temporal ratchet, ANM phenomenon can be induced in the absence of external signals.

In this paper, we investigate anomalous transport of an inertial particle in a symmetric periodic potential, with the phase of the potential being controlled by two cosine modulations, and study effect of the phase modulation on anomalous transport through its underlying physical mechanisms. The paper is constructed as follows: In Section 2, model and theoretical analysis are provided. In Section 3, results and discussions are presented. By means of numerical calculations, the transport induced by the phase modulation will be analyzed in detail. In Section 4, conclusions are drawn.

2. Model and theoretical analysis

The transport of an inertial particle in a spatially symmetric and time modulated periodic potential will be investigated. Under the influence of **an external force** F, the dynamics of the particle is governed by the following differential equation for the state variable x:

$$\ddot{x} + \gamma \dot{x} = -U'(x, t) + F + \xi(t), \tag{1}$$

γ is the friction coefficient, determining the **dynamics** of the system [48]. The dot above x represents the derivative of x with respect to time t, and $U'(x, t)$ denotes the partial derivative of $U(x, t)$ with respect to x. Here, $U(x, t)$ is a spatially time-modulated potential given by:

$$U(x, t) = -\frac{1.6}{k} \sin[kx + \varphi(t)]. \tag{2}$$

where k denotes the spatial frequency of the potential. Such a potential is a standing wave created experimentally by counterpropagating laser beams in order to build up a one-dimensional optical lattice potential for which its phase is modulated temporally [36]. Ref. [36] mentioned that stationary states of electrons in a lattice are plane waves modulated by periodic functions of position. In this model, $\varphi(t)$ can stand for a time-periodic modulation of standing wave position. External fields or disturbance may influence the modulation of the position to some degree. Therefore, the modulation function may not always be symmetric-periodic, but ratchet-periodic. Here, we assume that

$$\varphi(t) = \lambda_1 \cos(\beta_1 t) + \lambda_2 \cos(\beta_2 t + \phi), \tag{3}$$

λ_1 and λ_2 are the amplitudes of the periodic modulations, and β_1 and β_2 are the their frequencies. ϕ is the relative phase between the modulations. $\xi(t)$ is the Gaussian white noise with intensity D, satisfying the following statistical properties:

$$\langle \xi(t) \rangle = 0, \tag{4}$$

$$\langle \xi(t)\xi(t') \rangle = 2D\delta(t - t'). \tag{5}$$

All these physical quantities and parameters are in dimensionless form.

It is very difficult to obtain the current in the system (i.e., mean velocity of the particle) by analytical solution of Eq. (1). However, we can make use of a modified Euler algorithm to calculate the mean velocity (or current) numerically using its definition

$$\langle v \rangle = \lim_{t \longrightarrow \infty} [x(t) - x(0)]/t, \tag{6}$$