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A new fractal reliability model for networks with node fractal growth and no-loop

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HIGHLIGHTS

- A network classification based on fractal growth is put forward.
- A reliability model based on the fractal unit and its iterative process is proposed.
- The algorithm with an approximate complexity $O(|V|)$ is provided to calculate the model.
- The reliability of the widely investigated Koch network is studied and discussed.

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ABSTRACT

Evaluating the reliability of networked systems with existing exact or approximate methods often needs to characterize the detailed topology with node scale, which brings complexity and high computation effort. In this paper, a new reliability model based on the fractal unit with a bigger scale than nodes and a much smaller scale than whole network is proposed for networks with fractal growth and no-loop (NF-NL). The introduced model simplifies the K-terminal reliability (KTR) of a NF-NL network to a multiplication of different KTR of fractal units in the network. The corresponding algorithm is also given, which has a linear-time complexity $O(|V|)$ when the fractal unit scale is very small. Compared with the existing models, the proposed model provides a novel way to construct the reliability model only dependent on two factors: (1) the fractal unit characteristics and (2) its iterative process. Finally, the widely investigated Koch network case is studied with the proposed model.

1. Introduction

Reliability models such as fault tree and reliability block diagram [1,2], analyze a system failure by decomposing the system into different units, then using logic symbols or the diagram to characterize the failure mode of the units. These models are mostly used to analyze systems with simple and independent failures. With the rapid development of computer science and information technology, numerous networked systems are constructed and created in our daily life, such as transportation systems, power grids and the Internet of things. For these critical networked systems, the cascading failure, traffic dynamics and so on are proposed to investigate the systems behaviors and performance [3–7]. In existing much efforts, the efficient models and methods to evaluate the reliability of complex networked systems have always been concerned [8–16] (see Table 1).

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Table 1
Frequently used acronym and notation.

KTR	K-terminal reliability
NF	Node-fractal
NF-NL	Node-fractal & no-loop
NF-L	Node-fractal & loop
EF	Edge-fractal
AEF	All-edge-fractal
PEF	Partial-edge-fractal
$G(V, E)$	A graph G with a vertex set V and an edge set E
G_K	The graph G with a specified vertex set $K(K \subseteq V)$
$R(G_K)$	The K-terminal reliability of G
$G^F(V^F, E^F)$	A fractal unit graph G^F with a vertex set V^F and an edge set E^F
G^F_K	The fractal unit G^F with a specified vertex set $K(K \subseteq V^F)$
$R(G^F_K)$	The K-terminal reliability of G^F

The K-terminal reliability (KTR) is extensively used to evaluate the connectivity for networked systems. The exact method of enumeration based on the minimal pathsets or cutsets is proposed to calculate KTR of a network in early numerous literatures, where the inclusion–exclusion principle or the sum of disjoint is used [8–10]. In this method, a path is defined as a set of network components (the edges and/or vertices), and when they are all in work state, the system can work [11]. A path is minimal if it has no proper subpaths or the size of the path making system work is minimum. Conversely, a cut is a set of network components such that when they are all fail, the system is down [11]. And a cut is minimal if the size of failed components sets which make the system failure is minimum. The enumeration is proved NP-hard and unable to treat large networks (up to 20 vertices with low density) [11]. Methods based on graph transformation are proposed, where some reduction principles and factoring algorithms are used [12,13,17]. In these methods, the network is reduced by some rules, such as the series–parallel reduction [12] and the polygon-to-chain [13], which brings less computation efforts than the enumeration. However, it is also found that the computational time becomes prohibitive for large, or dense networks [11].

In order to reduce the complexity of reliability modeling and computation for large networks, some approximation methods are also proposed. Recursive truncation algorithm is introduced to estimate the connection reliability with a pre-specified accuracy [14]. This method first scans all minimal cutsets of the network, then finds some weak cutsets to gain an adaptive threshold [14]. Methods by using the set of subnetworks decomposed or branched from the given network are also proposed to calculate lower and upper bounds [15,18,19]. Furthermore, methods with an iterative procedure to obtain the lower and upper bounds of connection reliability are also used [16,20], which needs to enumerate partial mincuts or minpaths. Nevertheless, these methods also have high computational efforts when used to calculate the reliability for large network systems.

These existing exact and approximate methods have focused on evaluating network reliability by reducing network, or decreasing the pathset or cutset of a network. In these methods, the whole structure characteristic with a detailed description of the connections between the nodes and edges for the given system is captured in the performed process of reliability calculation, which leads to the complexity of modeling and computing for large networks. Thus, it is interesting to construct reliability model based on the fractal unit (the basic structure unit in a network) scale, which aims to make it easier and less computation efforts to evaluate network reliability. In this paper, a new fractal reliability model for networks with fractal growth and no-loop is proposed. The model is constructed based on the fractal unit which has a bigger scale than nodes or edges, and a much smaller scale than the network. In the introduced model, the KTR of a NF-NL network is expressed as a multiplication of different KTR of the fractal units in the network. The corresponding algorithm for the model is also proposed, which has a linear-time complexity $O(|V|)$ when the fractal unit scale is very small. Based on the model, the KTR for the famous and widely investigated Koch network is analyzed.

The remainder of this paper is organized as follows. In Section 2, the preliminaries used in this paper are first introduced. In Section 3, the network classification based on the fractal growth is proposed. In Section 4, the fractal reliability model for a kind of critical network of node-fractal and no-loop is put forward and the corresponding algorithm is also given. In Section 5, the Koch network is analyzed and discussed. Finally concluding remarks are provided in Section 6.

2. Preliminaries

Definition 1. Consider a graph $G = (V, E)$ where all vertices are perfectly reliable but any edge e_i may fail with probability q_i or work with probability $p_i = 1 - q_i$, and all edge failures occur independently of each other. Suppose a specified vertex set K in graph G. When certain vertices of V are specified to be in K, we denote the graph together with set K by G_K. Thus, the K-terminal reliability of G is denoted as $R(G_K)$, which means the probability that the K-vertices are connected, where all-terminal reliability and two-terminal reliability when $K = V$ and $|K| = 2$, respectively.

Definition 2 (*[12]*)**.** A cutvertex of a graph is a vertex whose removal disconnects the graph.