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Relaxing conditions for local average treatment effect in fuzzy regression discontinuity

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HIGHLIGHTS

- Two conditions are often assumed for LATE view of fuzzy regression discontinuity.
- 'Two-sided' monotonicity, and independence between running and potential variables.
- In this paper, the first assumption is relaxed to one-sided monotonicity.
- The second is relaxed to much weaker moment continuity conditions.

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ABSTRACT

In fuzzy regression discontinuity with a running/forcing variable S and a cutoff c, the identified treatment effect is the 'effect on compliers at $S = c$'. This well-known 'local average treatment effect (LATE)' interpretation requires (i) a monotonicity condition and (ii) the independence of the potential treatment and potential response variables from S. These assumptions can be violated, however, particularly (ii) when S affects potential variables, which can easily happen in practice. In this paper, we weaken both assumptions so that LATE in fuzzy regression discontinuity has a better chance to hold in the real world, and practitioners can claim their findings in fuzzy regression discontinuity to be LATE.

1. Introduction

Regression discontinuity (RD) is widely used for treatment effect analysis: see Imbens and Lemieux (2008), Lee and Lemieux (2010), Lee (2016), Choi and Lee (2017), Cattaneo and Escanciano (2017), Cattaneo et al. (2018), and references therein. In typical RD with a binary treatment $D = 0, 1$, an individual is assigned to the treatment or control group, depending on a running/forcing variable S crossing a cutoff c or not. Redefining S as $S - c$ to normalize the cutoff to 0, D then equals

$$\delta \equiv 1[0 \le S]$$

where $1[A] \equiv 1$ if A holds, and 0 otherwise. Here, D is fully determined by S, and such a RD is called a 'sharp RD'. If D is determined by S and some other random variables, then such a RD is 'fuzzy RD (FRD)'. As 'running/forcing variable' appears often in this paper, call it just 'score' (S for score) henceforth.

Let (D^0, D^1) be the potential treatments for $\delta = 0, 1$, and (Y^0, Y^1) the potential responses for $D = 0, 1$. Hahn et al. (2001) showed that the identified treatment effect parameter in FRD is

$$E(Y^1 - Y^0 | D^1 > D^0,\ S \simeq 0) = \frac{E(Y|0^+) - E(Y|0^-)}{E(D|0^+) - E(D|0^-)} \qquad (1.1)$$

under the monotonicity $D^1 \ge D^0$ *on a local neighborhood of* 0 *and* $(D^0, D^1, Y^0, Y^1) \amalg S$, where '$\amalg$' stands for independence, $E(Y|0^+) \equiv \lim_{s\downarrow 0} E(Y|S = s)$ and $E(Y|0^-) \equiv \lim_{s\uparrow 0} E(Y|S = s)$. The left-hand side of (1.1) is the 'effect on compliers at the cutoff', which is identified by the right-hand side ratio based on the observed (D, Y, S).

The complier interpretation of the FRD effect draws on the 'local average treatment effect (LATE)' of Imbens and Angrist (1994); as well known, LATE provides an attractive interpretation to instrumental variable estimator (IVE) in general. However, the monotonicity and independence assumptions in Hahn et al. (2001) for LATE in FRD are too restrictive: if S affects any of (D^0, D^1, Y^0, Y^1),

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then at least the independence assumption is violated, which most practitioners seem to be unaware of.

The goal of this paper is to weaken the monotonicity assumption on a local neighborhood of 0 *only to a local positive neighborhood of* 0*, and replace the independence condition for LATE in FRD with much weaker 'moment continuity' conditions of* $E(D^0|S)$ *and* $E\{(Y^1 - Y^0)D^0|S\}$ *at* $S = 0$, so that LATE in FRD has a better chance to hold in the real world and users of FRD can accord the LATE interpretation to their effect findings. Whereas the weakened monotonicity is hard to see at this stage, the following couple of examples demonstrate how easily the independence assumption is violated while the moment continuity conditions at $S = 0$ hold.

Suppose S is age, δ is a voting eligibility by S crossing the age limit, D is voting and Y is supporting the Republican party. In this example, S is likely to affect (D^1, Y^0, Y^1), because old persons tend to vote more than young persons when eligible (i.e., $E(D^1|S)$ is an increasing function of S) and tend to be conservative, i.e., $E(Y^1|S)$ and $E(Y^0|S)$ are increasing functions of S where $Y^1 = 1$ and $Y^0 = 1$ mean supporting the Republican party with and without voting. Another example is retirement eligibility δ, retirement D and consumption Y, where individuals are more likely to retire as they get older once they cross the age cutoff (i.e., $E(D^1|S)$ is an increasing function of age S), and $E(Y^1|S)$ and $E(Y^0|S)$ are decreasing functions of S because individuals consume less as they get older with and without retirement.

More specifically, for a violation of $(D^0, D^1) \amalg S$, consider

$$D^0 = 1[\alpha_0 + \alpha_s S + \varepsilon > 0], \quad D^1 = 1[\alpha_0 + \alpha_1 + \alpha_s S + \varepsilon > 0], \quad \alpha_1 \geq 0, \ \alpha_s > 0 \tag{1.2}$$

where the α's are parameters with $\alpha_1 \geq 0$ for the monotonicity, and ε is an error term with a continuous distribution function F symmetric about 0 with $\varepsilon \amalg S$. Here, '$(D^0, D^1) \amalg S$' does not hold except exactly at $S = 0$, but $E(D^0|S = s) = F(\alpha_0 + \alpha_s s)$ is continuous in s to allow the moment $E(D^0|S = s)$ to be a continuous function of s.

Similarly, for a violation of $(Y^0, Y^1) \amalg S$, consider

$$Y^0 = 1[\beta_0 + \beta_s S + U > 0], \quad Y^1 = 1[\beta_1 + \beta_s S + U > 0], \quad \beta_s \neq 0 \tag{1.3}$$

where the β's are parameters and U is an error term with a continuous distribution function G symmetric about 0 with $U \amalg (S, \varepsilon)$. It holds that

$$\begin{aligned} &E\{(Y^1 - Y^0)D^0|S = s\} \\ &= E\{(1[\beta_1 + \beta_s S + U > 0] - 1[\beta_0 + \beta_s S + U > 0]) \cdot D^0|S = s\} \\ &= E\{(1[\beta_1 + \beta_s s + U > 0] - 1[\beta_0 + \beta_s s + U > 0]) \\ &\quad \cdot 1[\alpha_0 + \alpha_s s + \varepsilon > 0]\} \\ &= \{G(\beta_1 + \beta_s s) - G(\beta_0 + \beta_s s)\} \cdot F(\alpha_0 + \alpha_s S) \quad \text{(due to } U \amalg \varepsilon) \end{aligned}$$

which is continuous in s, but '$(Y^0, Y^1) \amalg S$' does not hold except exactly at $s = 0$.

The above (D^0, D^1) and (Y^0, Y^1) models are very general, because the assumptions on (ε, U) are imposed just to simplify our exposition, and as such, they can be relaxed without difficulty. Note how easily the independence breaks down by S entering the potential variable models. Also note that, in (1.2) and (1.3), the continuity of $E(D^0|S = s)$ and $E\{(Y^1 - Y^0)D^0|S = s\}$ holds at all values of s, not just at $s = 0$.

The remainder of this paper is organized as follows. Section 2 reviews the aforementioned conditions in Hahn et al. (2001) for (1.1). Section 3, which is the main contribution of this paper, weakens the conditions as stated above and discusses the related literature. Finally, Section 4 concludes our findings.

2. Two-Sided monotonicity and independence

Take $E(\cdot|0^+)$ and $E(\cdot|0^-)$ on the observed response $Y = Y^0 + (Y^1 - Y^0)D$ to get

$$\begin{aligned} E(Y|0^-) &= E(Y^0|0^-) + E\{(Y^1 - Y^0)D|0^-\}, \\ E(Y|0^+) &= E(Y^0|0^+) + E\{(Y^1 - Y^0)D|0^+\}. \end{aligned} \tag{2.1}$$

If $Y^1 - Y^0 = \beta_d$, a constant, then *assuming the continuity of* $E(Y^0|S = s)$ *at* 0, β_d becomes the ratio in (1.1), which can easily be seen by differencing the two equations in (2.1). Hahn et al. (2001) went further to allow

$$Y^\Delta \equiv Y^1 - Y^0$$

to be a random variable as follows.

Replace the two D's in (2.1) with D^0 and D^1, respectively, and take the difference of the two equations in (2.1) to obtain

$$E(Y|0^+) - E(Y|0^-) = E(Y^\Delta D^1|0^+) - E(Y^\Delta D^0|0^-). \tag{2.2}$$

The right-hand side has two terms conditioned on different sets. To overcome this problem, Hahn et al. (2001) invoked the *independence assumption* $(Y^\Delta, D^0, D^1) \amalg S$ *on* $S \in (-\nu, \nu)$ *for some* $\nu > 0$, adopting the independence assumption '$(Y^0, Y^1, D^0, D^1) \amalg$ instrument' in Imbens and Angrist (1994) for IVE in general. Then (2.2) becomes

$$\begin{aligned} &E(Y|0^+) - E(Y|0^-) \\ &= E\{Y^\Delta D^1|S \in (-\nu, \nu)\} - E\{Y^\Delta D^0|S \in (-\nu, \nu)\} \\ &= E\{Y^\Delta (D^1 - D^0)|S \in (-\nu, \nu)\} \\ &= E\{Y^\Delta|D^1 - D^0 = 1, \ S \in (-\nu, \nu)\} \\ &\quad \cdot P\{D^1 - D^0 = 1|S \in (-\nu, \nu)\} \end{aligned} \tag{2.3}$$

under the *monotonicity assumption* $D^1 \geq D^0$ *on* $S \in (-\nu, \nu)$ to rule out $D^1 - D^0 = -1$.

Using '$(D^0, D^1) \amalg S$ on $S \in (-\nu, \nu)$' throughout, we have

$$\begin{aligned} &E(D|0^+) - E(D|0^-) = P(D^1 = 1|0^+) - P(D^0 = 1|0^+) \\ &= \{P(D^1 = 1, D^0 = 1|0^+) + P(D^1 = 1, D^0 = 0|0^+)\} \\ &\quad - P(D^1 = 1, D^0 = 1|0^+) \\ &= P(D^1 = 1, D^0 = 0|0^+) = P\{D^1 - D^0 = 1|S \in (-\nu, \nu)\}. \end{aligned}$$

The first and last terms give

$$P\{D^1 - D^0 = 1|S \in (-\nu, \nu)\} = E(D|0^+) - E(D|0^-). \tag{2.4}$$

Finally, (2.4) and *the D-break assumption* $E(D|0^+) - E(D|0^-) \neq 0$ applied to the last term of (2.3) render

$$E\{Y^\Delta|D^1 - D^0 = 1, \ S \in (-\nu, \nu)\} = \frac{E(Y|0^+) - E(Y|0^-)}{E(D|0^+) - E(D|0^-)} \tag{2.5}$$

which is what Theorem 3 in Hahn et al. (2001) essentially states.

3. One-Sided monotonicity and moment continuity

Maintaining the continuity assumption of $E(Y^0|S)$ in Hahn et al. (2001) invoked for (2.1) to (2.2), impose the *continuity assumption* of $E(Y^\Delta D^0|S)$ and $E(D^0|S)$ at $S = 0$, instead of the independence $(Y^\Delta, D^0, D^1) \amalg S$ on $S \in (-\nu, \nu)$:

$$\text{(i)}: E(Y^\Delta D^0|0^+) = E(Y^\Delta D^0|0^-) \quad \text{and} \quad \text{(ii)}: E(D^0|0^+) = E(D^0|0^-). \tag{3.1}$$

Also assume that the following right-limits exist, which is hardly a restriction:

$$\lim_{s \downarrow 0} E(Y^\Delta|D^1 - D^0 = 1, \ S = s) \quad \text{and} \quad \lim_{s \downarrow 0} P(D^1 - D^0 = 1|S = s). \tag{3.2}$$