



Designing efficient and incentive compatible mechanisms is almost impossible in quasi-linear environments[☆]



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ABSTRACT

In quasi-linear environments, classic theories state that it is possible to design efficient and incentive-compatible mechanisms, such as Vickrey, Clarke and Groves (VCG) mechanisms. However, once financial constraints are taken into account, we find that almost no financial constraint is compatible with efficiency and individual incentives over unrestricted domains and some restricted domains. Therefore, our results imply that even in quasi-linear environments, it is still impossible to design an efficient and incentive compatible mechanism because of financial constraints.

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1. Introduction

In quasi-linear environments without external funding, Vickrey (1961), Clarke (1971) and Groves (1973) provided mechanisms, known as Groves or VCG mechanisms that induce truthful revelation of preferences and lead to efficient decisions. Green and Laffont (1979) show that the converse of the proposition above is also true in the following sense: if the domain is rich enough, then a mechanism that induces truthful revelation of preferences leading to efficient decisions must be a VCG mechanism. However, another important result of Green and Laffont (1979) is that if the set of agents' types is sufficiently rich (so that agents may hold any payoff function), then no VCG mechanism is budget balanced. In other words, any VCG mechanism is not budget balanced, namely, the agents' payments will sum to less than 0. This means the agents must accept some waste of the transferable commodity or loss of efficiency. Cavallo (2006), Guo and Conitzer (2009); Mehta et al. (2009); Moulin (2010) make attempts to estimate and minimize

the budget imbalance. Yi and Li (2016) conclude that there is no VCG mechanism whose worst absolute loss of efficiency is finite over unrestricted domains. In addition, Moulin (2009) studies the relative boundedness of budget.

In this paper, we consider a class of general financial constraints of budget in which each constraint is given by a function that assigns a bound on the loss of efficiency to any profile of individuals' utilities. It is easy to check that our new budget constraints are weaker than the following requirements: budget balance (Green and Laffont, 1979; Walker, 1980), absolute budget boundedness (Yi and Li, 2016), individuals' budget constraint (Che and Gale, 1998; Laffont and Robert, 1996; Maskin, 2000; Andersson and Svensson, 2014, 2016), individual rationality requirement, and relative boundedness of budget (Moulin, 2009).

Our main result is also an impossibility: in classic quasi-linear environments, there is no incentive-compatible and efficient mechanism that satisfies budget feasibility with respect to some constraint function over both unrestricted domains and some specific restricted domains. Our impossibility theorem reveals that it is impossible to design incentive-compatible and efficient mechanisms in quasi-linear environments because budget constraints always exist in practical application. Furthermore, we prove that there is no efficient and incentive compatible mechanism satisfying (absolutely) budget boundedness over positive domains, which extends the results of Yi and Li (2016) to a class of restricted

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domains. Note that, our impossibility theorems imply: there does not exist an efficient and incentive compatible mechanism that satisfies absolutely or relative budget boundedness over both unrestricted and some restricted domains.

This paper is organized as follows: Section 2 presents the social choice model. Section 3 proves our impossible theorems.

2. The model

Consider a society with n agents, denoted by $I = \{1, 2, \dots, n\}$. The set of potential outcomes is denoted by A which can be a finite or infinite set. Each $x \in A$ can be viewed as a public project. For our purposes, we assume that $n \geq 2$ and $\#(A) \geq 2^1$. For every agent i , he has a set of valuation functions denoted by $V_i \subset \mathcal{U}(A, R)$, where $\mathcal{U}(A, R)$ is the set of all functions from A to $R = (-\infty, +\infty)$. For each agent i , i 's valuation function is his private information. We denote a profile of valuation functions as $v = (v_1, v_2, \dots, v_n)$ and the product of valuation function spaces of all agents as $V = V_1 \times V_2 \times \dots \times V_n$. For convenience, given $v, v' \in V$ and $S \subset I, (v_S, v'_S)$ denotes a profile in which an agent i has valuation function v_i if $i \in S$ and v'_i if $i \notin S$. Particularly, when $S = \{i\}$, we shall write (v_i, v'_i) rather than $(v_{\{i\}}, v'_{\{i\}})$.

When transfers are allowed, an alternative is then a vector $(x, y_1, y_2, \dots, y_n)$, where x is an element of A and $y_i \in (-\infty, +\infty)$ is a transfer of a numeraire to agent i . We assume that agent i 's utility depend on an outcome $x \in A$, and a transfer payment y_i in a quasilinear manner: $v_i(x) + y_i$.

A decision rule is a function f from V to A that assigns a unique collective choice $f(v)$ to each possible profile $v = (v_1, v_2, \dots, v_n)$. A transfer is vector function $t = (t_1, t_2, \dots, t_n): V \rightarrow R^n$. The function $t_i(v)$ represents the payment that agent i receives (or loses if it is negative) based on the announcement of types v . A social choice function or mechanism in this quasilinear environment takes the form (f, t) where f is a decision rule and t a transfer function.

We consider a closed system in which there is no source of outside funding for the agents. In this case, t satisfies $\sum_{i=1}^n t_i(v) \leq 0$. A transfer t is budget balanced if $\sum_{i=1}^n t_i(v) = 0$ for all $v \in V$. If $\sum_{i=1}^n t_i(v)$ is less than zero, then it generates a surplus that would have to be wasted.

A social choice function (f, t) is decisively efficient or f is efficient if $\sum_{i=1}^n v_i(f(v)) \geq \sum_{i=1}^n v_i(x)$ for all $v \in V$ and all $x \in A$.

A social choice function (f, t) is incentive compatible if, for all $i \in I$ and all $v \in V$,

$$v_i(f(v)) + t_i(v) \geq v_i(f(v'_i, v_{-i})) + t_i(v'_i, v_{-i})$$

for all v'_i . A decision rule f is implementable if there exists a transfer function t such that (f, t) is incentive-compatible.

3. Main results

In quasi-linear environments, for any efficient decision rule, a transfer t exists, $t_i(v) = \sum_{j \neq i} v_j(f(v)) + h_i(v_{-i})$, such that (f, t) is incentive-compatible (Vickrey, 1961; Clarke, 1971; Groves, 1973). Under certain conditions, we can restrict $\sum_i t_i(v) \leq 0$, that is, there is no external fund inflow.² This is the well-known VCG mechanism. Moreover, under certain mild assumptions on the richness of the domain (for example, the following Assumption A), the VCG mechanism is the only one that has these properties (see Green and Laffont, 1979; Holmstrom, 1979). However, under unrestricted domains, Green and Laffont (1979) show that VCG mechanism does not satisfy budget-balance and hence there is waste

or loss of efficiency. Yi and Li (2016) prove further that there is no decisively efficient and incentive compatible mechanism (f, t) that satisfies budget-boundedness. It means that VCG mechanism is infeasible in practice if the maximum loss that a society can afford is $W_0 \in (0, +\infty)$. However, some may argue that the maximum loss that the society can afford could be $W_0 + \varphi(\sum_{i \in I} v_i(f(v)))$ rather than W_0 , where $\sum_{i \in I} v_i(x)$ is the total net benefit from use of the project x and $\varphi(\sum_{i \in I} v_i(f(v)))$ represents the present value of $\sum_{i \in I} v_i(f(v))$. Thus, the budget constraint is

$$-\varphi\left(\sum_{i \in I} v_i(f(v))\right) - \Omega_0 \leq \sum_{i \in I} t_i(v).$$

With this budget constraint, VCG mechanism might work.

In this paper, we consider a general budget constraint:

$$\Phi(v_1(f(v)), v_2(f(v)), \dots, v_n(f(v))) \leq \sum_{i \in I} t_i(v)$$

for all $v \in V$, where $\Phi: R^n \rightarrow (-\infty, 0]$. We say that (f, t) satisfies budget feasibility with respect to Φ if the inequality of budget constraint is satisfied. Obviously, if one mechanism satisfies any of the following constraints: budget-balance, budget-boundedness, individual budget constraint, individual rationality or relative boundedness of budget, then it must satisfy budget feasibility with respect to some Φ . The question of interest here is that for what constraint function $\Phi: R^n \rightarrow (-\infty, 0]$, is it possible to design efficient and incentive compatible mechanisms that satisfy financial feasibility with respect to Φ ? Unfortunately, our conclusion is negative: for any constraint function $\Phi: R^n \rightarrow (-\infty, 0]$, it is impossible to design decisively efficient and incentive compatible mechanisms that satisfy budget feasibility with respect to Φ .

Before we prove our impossibility theorem, we first establish a lemma regarding revenue equivalence.

Given i and v_i , let v_i^k be defined by $v_i^k(x) = \frac{k}{m} v_i(x)$ for $k = 0, 1, 2, \dots, m$, and let

$$T_i(v_i, v_{-i}) = \lim_{m \rightarrow +\infty} \sum_{k=0}^{m-1} [v_i^{k+1}(f(v_i^{k+1}, v_{-i})) - v_i^{k+1}(f(v_i^k, v_{-i}))].$$

We see in the proof of Lemma 1 that the limit of the sequence $\sum_{k=0}^{m-1} [v_i^{k+1}(f(v_i^{k+1}, v_{-i})) - v_i^{k+1}(f(v_i^k, v_{-i}))]$ always exists whenever (f, p) is incentive-compatible.

We say that V_i is a cone if $rv_i \in V_i$ when $v_i \in V_i$ and $r \geq 0$, and V_i is a double-cone if $v_i \in V_i$ implies $rv_i \in V_i$ for every $r \in (-\infty, +\infty)$. Obviously, a double-cone is a cone.

Lemma 1. Suppose that, for every $i \in I$, V_i is a cone. If (f, t) is incentive-compatible, then, for each i ,

$$t_i(v_i, v_{-i}) = T_i(v_i, v_{-i}) + t_i(0, v_{-i}) \text{ for any } v,$$

$$\text{where } T_i(v_i, v_{-i}) = \lim_{m \rightarrow +\infty} \sum_{k=0}^{m-1} [v_i^{k+1}(f(v_i^{k+1}, v_{-i})) - v_i^{k+1}(f(v_i^k, v_{-i}))].$$

Proof. Let $v_i^k = \frac{k}{m} v_i (k = 0, 1, 2, \dots, m)$. Then $v_i^0 = 0$ and $v_i^m = v_i$. By incentive compatibility,

$$\begin{aligned} v_i^k(f(v_i^k, v_{-i})) + t_i(v_i^k, v_{-i}) &\geq v_i^k(f(v_i^{k+1}, v_{-i})) + t_i(v_i^{k+1}, v_{-i}) \\ v_i^{k+1}(f(v_i^{k+1}, v_{-i})) + t_i(v_i^{k+1}, v_{-i}) &\geq v_i^{k+1}(f(v_i^k, v_{-i})) \\ &\quad + t_i(v_i^k, v_{-i}) \end{aligned}$$

Thus

$$\begin{aligned} v_i^k(f(v_i^k, v_{-i})) - v_i^k(f(v_i^{k+1}, v_{-i})) &\geq t_i(v_i^{k+1}, v_{-i}) - t_i(v_i^k, v_{-i}) \\ &\geq v_i^{k+1}(f(v_i^k, v_{-i})) - v_i^{k+1}(f(v_i^{k+1}, v_{-i})). \end{aligned}$$

¹ $\#(A)$ denotes the number of elements in A .

² For example, A is finite.

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