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A novel orthogonalized fractional order filtered-x normalized least mean squares algorithm for feedforward vibration rejection

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A B S T R A C T

This paper mainly discusses a pattern of vibration rejection by using a feedforward controller. Conventional filtered-x normalized least mean squares (Fx-NLMS) algorithm converges slow especially when signals are not independent on each other. Therefore, a novel orthogonalized fractional order filtered-x normalized least mean squares (Orth-FONLMS) algorithm is proposed to obtain the precise parameters of the feedforward controller. The proposed algorithm is an improved Fx-NLMS algorithm by using fractional calculus and orthogonal transform. To implement the algorithm, fractional order gradient method and adaptive lattice filter are introduced, respectively. Numerical simulation proves that the proposed algorithm outperforms over the Fx-NLMS and fractional order filtered-x normalized least mean squares (Fx-FONLMS) algorithm in terms of convergence rate and accuracy.

1. Introduction

Disturbance rejection has been an important issue in control and signal processing field [1–4]. Disturbance can be interpreted as vibration or noise, which lead to vibration rejection and noise cancellation [5–8]. As for vibration rejection, it is noted that the controller can be designed either in feedback or feedforward architecture. In [9], the case of hard drive disk vibration rejection with a feedback sensor was introduced. An example of bandwidth vibration rejection with a feedforward controller was given in [10]. According to the researches mentioned, it is known that if the vibration can be obtained, then the feedforward controller can be effectively designed. Besides, feedforward control does not change the performance of the original closed-loop control system. In this case, the Fx-NLMS algorithm and some modified LMS algorithms are considered as good ways to gain the parameters of controllers, so that they are widely used both in vibration rejection and noise cancellation [11–16]. However, traditional Fx-LMS and Fx-NLMS algorithms have some shortcomings even though they are effective in many aspects. One of the most crucial flaws is that they converge slow particularly when the signals are relevant to each other.

In order to enhance the performance of algorithms, various methods were introduced. Orthogonal transform was commonly used to speed up the convergence rate of Fx-LMS algorithm [17–20]. Many forms of orthogonalization have been applied to different study fields in recent researches, such as discrete cosine transform [21], discrete sine transform [22], dis-

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crete Fourier transform [23] and adaptive lattice filter [10]. Among all these methods of orthogonalization, adaptive lattice filter needs comparably less computational complexity and suits better on the issue we discussed. After orthogonalization, signals are independent on each other which obviously simplify the classic algorithms. Moreover, it is noted that fractional calculus is becoming more and more influencing in improving adaptive filtering algorithms. In [24], a novel fractional order gradient descent method was proposed, which could be applied to various scenarios. [25,16] developed a fractional order LMS algorithm to improve performances in terms of convergence rate and steady state error. Fractional calculus was also used to gain good performances in system identification [26], automatic control [27], fault diagnosis [28] and artificial neural networks [29]. All these work mentioned above have demonstrated that the orthogonal transform and fractional calculus are helpful for improving performances of algorithms.

Inspired by discussions above, the aims of this paper are: 1) establishing a feedforward vibration rejection scheme and a feedforward controller model; 2) using fractional order gradient descent method to modify conventional Fx-NLMS algorithm to form Fx-FONLMS algorithm; 3) introducing adaptive lattice filter as a method of orthogonalization into Fx-FONLMS algorithm to propose the Orth-FONLMS algorithm for feedforward vibration rejection.

The rest of this paper is organized as follows. Firstly, a brief introduction of fractional calculus and a control model of vibration rejection problem are given in Section 2. Conventional Fx-NLMS algorithm is also introduced in this section. Secondly, main results are shown in Section 3. Optimal solution of controller parameters are derived, fractional order gradient descent method and adaptive lattice filter are introduced into traditional Fx-NLMS algorithm, computational complexity and convergence analysis are also given. In this section, Fx-FONLMS and Orth-FONLMS are proposed, respectively. And then, numerical simulation is carried out in Section 4. At last, Section 5 draws the conclusions.

2. Preliminaries

This section gives some necessary information about our work. Subsection 2.1 presents basic mathematical background of fractional calculus, which will be used to design the fractional order update equation of the proposed algorithm, 2.2 offers the considered vibration rejection model and 2.3 introduces the conventional Fx-NLMS algorithm to solve this kind of problem.

2.1. Fractional calculus

Before the discussion, it is necessary to present the mathematical background of the fractional calculus. There are three kinds of prevalently used definitions of fractional calculus including Grünwald-Letnikov, Riemann-Liouville and Caputo, respectively [30].

The Riemann-Liouville definition of α-order derivative, for a given function $f(t)$, is expressed as

$$ {}^{\mathrm{RL}}_{t_0}\mathcal{D}^{\alpha}_{t} f(t) = \frac{1}{\Gamma(n-\alpha)} \frac{\mathrm{d}^n}{\mathrm{d}t^n} \int_{t_0}^{t} \frac{f(\tau)}{(t-\tau)^{\alpha-n+1}} \mathrm{d}\tau, \tag{1} $$

where $n-1 < \alpha < n$, $n \in \mathbb{N}^+$. As for this paper, the Riemann-Liouville definition of derivative is adopted. ${}^{\mathrm{RL}}_{t_0}\mathcal{D}^{\alpha}_{t}$ will be abbreviated, for convenience, as ${}_{t_0}\mathcal{D}^{\alpha}_{t}$ in the following parts.

If function $f(t)$ can be expanded as Taylor series at $t = t_0$, the Riemann-Liouville fractional derivative (1) can be rewritten as follows [31]

$$ {}_{t_0}\mathcal{D}^{\alpha}_{t} f(t) = \sum_{i=0}^{+\infty} \frac{f^{(i)}(t)}{\Gamma(i+1-\alpha)} (t-t_0)^{i-\alpha}, \tag{2} $$

where $n-1 < \alpha < n, n \in \mathbb{N}^+$. This series representation in (2) is a crucial equation of fractional order gradient descent method [24].

Remark 1. It should be noted that although the Riemann-Liouville definition is adopted in this paper, all the three definitions of fractional calculus are applicable to design the orthogonalized fractional order Fx-NLMS algorithm developed in this paper.

2.2. Vibration rejection

Fig. 1 shows a common single input single output system. G and C_F constitute a stable closed-loop system, where G denotes a linear time invariant (LTI) system and C_F indicates an LTI feedback compensator. C_A represents an adaptive feedforward controller, whose function is to compensate the disturbance $\bar{v}$. n is the broad-band disturbance which can be attenuated by C_F. a is the measurement of v passing through a sensor S_D. It is confirmed that the design of feedforward controller is identical whether the vibration is at the output side or input side [10]. Supposing G and C_F are discrete time systems, according to Fig. 1