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On a robust and accurate localized artificial diffusivity scheme for the high-order flux-reconstruction method

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A B S T R A C T

The high-order flux reconstruction (FR) method on unstructured hexahedral grids is coupled with the localized artificial diffusivity (LAD) scheme for aiming at accurate simulation of shock-turbulence interaction. In order to overcome known robustness issues particularly with the high-order ($r > 0$) derivative formulation, important properties of the artificial bulk viscosity (ABV) profile affecting the solution are investigated first. For the purpose of comparison, an approximated Gaussian filter and modified restriction-prolongation (RP) filters are developed and tested for the present FR-LAD approach. Then, we propose a multidimensional RP filter on unstructured quadrilateral and hexahedral grids to address the issues on non-Cartesian grids. It is shown that a simple extension of a one-dimensional RP filter for two-dimensional grids may result in insufficient smoothing of ABV with $r = 2$, and that the proposed multidimensional RP filter can provide smooth ABV with both $r = 0$ and $r = 2$. The proposed FR-LAD scheme is tested for typical shock-related problems, including the 1D shock tube, 1D shock-entropy wave interaction, 2D steady shock flows, and 2D shock-vortex interaction. The FR-LAD scheme has favorable properties of subcell shock capturing with the length scale of $O(h/p)$ and superior preservation of high-order accuracy for smooth flows. Finally, LES of an overexpanded supersonic jet is performed to demonstrate its capability for practical applications. The proposed FR-LAD scheme can be an attractive candidate for LES or DNS of compressible flows involving shocks, contact discontinuities, turbulence, and their interactions on unstructured meshes.

1. Introduction

High-order methods on unstructured grids have been developed over the past three decades and are now expected to enable large eddy simulation (LES) for engineering applications with complicated geometries. Intensive research and development efforts on high-order unstructured grid methods are mainly led by the successful development of the discontinuous Galerkin (DG) method [1,2]. This study focuses on the high-order flux reconstruction (FR) method [3], also known as the correction procedure via reconstruction (CPR) method [4], which is a comprehensive framework of other unstructured high-order methods such as a nodal DG method [1,5] and the spectral difference method [6] or multidomain staggered grid method [7]. One of the critical issues regarding high-order unstructured grid methods is the lack of a robust shock capturing technique that can maintain high accuracy away from the discontinuities. When higher order approximation is used,

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some kind of dissipation must be added in order to obtain a stable solution in the presence of numerical discontinuities. However, this could adversely affect the solution accuracy, especially the resolution of turbulent scales. Thus, the development of numerical algorithms that capture discontinuities without spurious oscillations and also resolve broadband scales of turbulence remains a significant challenge.

Discontinuous capturing schemes are roughly categorized into two types of approaches. One is associated with a nonlinear limiter to detect discontinuities and control spurious oscillations near such discontinuity. The straightforward approach entails reducing the order of the interpolating polynomial there. Inspired by the finite volume methodology, total variation bounded (TVB) type slope limiters [8,9] and more elaborate moment limiters [10–12] have been proposed. Although these limiters have been very successful in the literature, they can degrade accuracy when used in smooth regions of the solution. And sometimes these limiters have also suffered worse convergence for steady-state problems, possibly due to the inconsistency between the original high-order reconstruction and the limited one. More sophisticated weighted essentially non-oscillatory (WENO) type schemes [13,14] allow for stable discretization near discontinuities, while maintaining a high-order approximation. However, such schemes have yet to be demonstrated in a practical 3D unstructured mesh context using the high-order reconstruction.

Another type of approach is based on adding artificial viscosity. One common approach is to use the Laplacian terms in the governing equations. Persson and Peraire [15] introduced an element-wise constant artificial viscosity combined with a discontinuous sensor and demonstrated the subcell shock resolution with the length scale of $O(h/p)$. This approach has been shown to be effective for DG methods [16,17]. In [15], artificial shear viscosity that augments the physical diffusion in the Navier–Stokes equations was also used in obtaining a sharper shock profile compared to the Laplacian approach. Recently, Asthana et al. [18] extended the spectral viscosity [19] that is introduced by Laplacians for a FR method and showed a shock being captured within one cell using a very high-order polynomial of $p = 119$. These approaches based on artificial viscosity are apparently effective in stabilizing computations with discontinuities. However, the performance against turbulence-shock interaction has yet to be fully investigated. There is a study that shows how adding artificial shear viscosity damps not only shocks but also turbulence away from the shocks [20,21].

The localized artificial diffusivity (LAD) method originally proposed by Cook and Cabot [22] and Cook [23] offers an attractive alternative means of accurately simulating such flows. The main feature of this method is to locally add artificial diffusivities to capture different types of discontinuities, such as shock, contact and material discontinuities. In particular, by introducing artificial bulk viscosity (ABV) for the shock waves, only the dilatational motion is damped to capture shocks numerically while minimally affecting the vortical structures, and thus ABV is well suited for shock-turbulence interaction problems. This approach coupled with a high-order compact differencing scheme has demonstrated desirable performance for LES of compressible turbulent flows with shocks [24,20]. A basic formulation of LAD for ABV is written in the following form:

$$\beta^* \sim \rho \Delta^{r+2} \overline{\left|\nabla^r F_\beta\right|}, \tag{1}$$

where ρ is the density, Δ is the grid spacing and F_β is a sensor function usually taken as magnitude of the strain rate or dilatation. r is taken as even numbers and ∇^r is decomposed into a series of Laplacians. The overbar denotes a smoothing filter to remove high wavenumber oscillations introduced by the Laplacians and the absolute value operator. For structured mesh methods, direct methods of evaluating the Laplacians and the Gaussian filter are well established. In the context of an unstructured mesh, extending a stencil to evaluate these quantities is not straightforward, however, and thus conventional techniques in the structured meshes cannot be applied directly. LAD has recently been extended for the spectral difference (SD) method [25,26], the flux reconstruction (FR) method [27], and the discontinuous Galerkin method [28]. Although the developed schemes were successfully applied to typical shock-related benchmark cases, the application of LAD remains limited to relatively simple problems due to the robustness issues reported there. In [26], the LAD formulation with $r = 2$ was reported to cause numerical instability on non-Cartesian girds (i.e., curvilinear or fully unstructured grids). A possible reason for the lack of robustness is considered to be the computed non-smooth artificial diffusivity. Yu and Yan [28] developed a high-order derivative scheme by ensuring second order accuracy and successfully applied the $r = 2$ form to a DG method, although the results were limited to structured quadrilateral grids and regular triangle grids generated by splitting the Cartesian grids. Mani et al. [24] showed that the use of $r = 0$ formulation damps sound propagation, especially at high frequency, and thus high-order formulation $r > 0$ is crucial for the prediction of sound propagation.

In this study, we aim to realize a robust and accurate shock capturing FR method using LAD in particular with $r = 2$. To accomplish this goal, we first revisit the high-order derivative evaluation for the discontinuous high-order methods including DG, SD, and FR. Emphasis is placed on the derivative calculation on unstructured grids. In addition, we clarify the causes of the robustness issues related to the conventional smoothing filter for LAD and its extension for multidimensions, and then propose improvements of the smoothing filter.

The following section first gives a brief review of the FR method in the strong conservation form. Then the formulation of LAD is defined, followed by the description of a suitable high-order derivative evaluation and an improved multidimensional smoothing filter in the context of the FR method. Numerical results for typical shock related flows are obtained to show the superior performance of the proposed FR-LAD scheme. Finally, LES of an overexpanded supersonic jet is performed to demonstrate its capability for large-scale practical applications.