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Constrained energy minimization based upscaling for coupled flow and mechanics

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ABSTRACT

In this paper, our aim is to present (1) an embedded fracture model (EFM) for coupled flow and mechanics problem based on the dual continuum approach on the fine grid and (2) an upscaled model for the resulting fine grid equations. The mathematical model is described by the coupled system of equation for displacement, fracture and matrix pressures. For a fine grid approximation, we use the finite volume method for flow problem and finite element method for mechanics. Due to the complexity of fractures, solutions have a variety of scales, and fine grid approximation results in a large discrete system. Our second focus in the construction of the upscaled coarse grid poroelasticity model for fractured media. Our upscaled approach is based on the nonlocal multicontinuum (NLMC) upscaling for coupled flow and mechanics problem, which involves computations of local basis functions via an energy minimization principle. This concept allows a systematic upscaling for processes in the fractured porous media, and provides an effective coarse scale model whose degrees of freedoms have physical meaning. We obtain a fast and accurate solver for the poroelasticity problem on a coarse grid and, at the same time, derive a novel upscaled model. We present numerical results for the two dimensional model problem.

0. Introduction

In the reservoir simulation, mathematical modeling of the fluid flow and geomechanics in the fractured porous media plays an important role. A coupled poroelastic models can help for better understanding of the processes in the fractured reservoirs. In this work, we consider an embedded fracture model (EFM) for coupled flow and mechanics problems based on the dual continuum approach. The mathematical model is described by the coupled system of equations for displacement and fracture/matrix pressures [35]. Coupling of the fracture and matrix equations is derived from the mass exchange between the two continua (transfer term) and based on the embedded fracture model. For the geomechanical effect, we consider deformation of the porous matrix due to pressure change, where pressure plays a role of specific source term for deformation [5,6,27–30]. Fundamentally, the system of equations is coupled between flow and geomechanics, where dis-

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placement equation includes the volume force, which is proportional to the pressure gradient, and the pressure equations include the term, which describes the compressibility of the medium.

Fracture networks commonly have complex geometries with multiple scales, and usually have very small thickness compared to typical reservoir sizes. Due to high permeability, fractures have a significant impact on the flow processes. A common approach to the fracture modeling is to model them as lower dimensional problems [13,14,21,34]. The result is a coupled mixed dimensional flow models, where we consider flow in the two domains (matrix and fracture) with mass transfer between them. In this work, the fractures are not resolved by grid but included as an overlaying continuum with an exchange term between fracture and matrix that appears as an additional source (Embedded Fracture Model (EFM)) [24,36,37]. This approach is related to the class of multicontinuum model [3,15,39]. Instead of the dualcontinuum approach, we represent fractures directly using lower dimensional flow model embedded in a porous matrix domain. In EFM, we have two independent grids for fracture networks and matrix, where simple structured meshes can be used for the matrix.

For geomechanics, we derive an embedded fracture model, where each fracture provides an additional source term for the displacement equation. This approach is based on the mechanics with dual porosity model [42,45]. In this model, we suppose displacement continuity on the fracture interface. For the discrete fracture model, a specific enrichment of the finite element space can be used for accurate solution of the elasticity problem with displacement discontinuity [2]. In this paper, we focus on the fully coupled poroelastic model for embedded fracture model and construct an upscaled model for fast coarse grid simulations. For the fine grid approximation, we use the finite volume method (FVM) for flow problem and the finite element method (FEM) for geomechanics. FVM is widely used as discretization for the simulation of flow problems [4,38]. We use a cell centered finite volume approximation with two point flux approximation (TPFA) for pressure. FEM is typically used for approximating the solid deformation problem. We use a continuous Galerkin method with linear basis functions with accurate approximation of the coupling term.

Fine grid simulation of the processes in fractured porous media leads to very expensive simulations due to the extremely large degrees of freedoms. To reduce the cost of simulations, multiscale methods or upscaling techniques are used, for example, in [16,25,26,33,41]. In our previous works, we presented multiscale model reduction techniques based on the Generalized multiscale finite element method (GMsFEM) for flow in fractured porous media [1,9,19]. In GMsFEM approach, we solve a local spectral problem for the multiscale basis construction [7,8,17,18]. This gives us a systematic way to construct the missing degrees of freedom via multiscale basis functions. In this work, we construct an upscaled coarse grid poroelasticity model with embedded fracture model. Our approach uses the general concept of nonlocal multicontinua (NLMC) upscaling for flow [10,11] and significantly generalized it to the coupled flow and mechanics problems. The local problems for the upscaling involves computations of local basis functions via an energy minimization principle and the degrees of freedom are chosen such that they represent physical parameters related to the coupled flow and mechanics problem. We summarize below the main goals of our work:

- a new fine grid embedded fracture model for poroelastic media (coupled system),
- a new accurate and computationally effective fully coarse grid model for coupled multiphysics problem using NLMC whose degrees of freedoms have physical meaning on the coarse grid.

Nonlocal multicontinua (NLMC) upscaling for processes in the fractured porous media provides an effective coarse scale model with physical meaning, and leads to a fast and accurate solver for coupled poroelasticity problem. To capture fine scale processes at the coarse grid model, local multiscale basis functions are presented. Constructing the basis functions based on the constrained energy minimization problem in the oversampled local domain is subject to the constraint that the local solution vanishes in other continua except the one for which it is formulated. Multiscale basis functions have spatial decay property in local domains and separate background medium and fractures. The proposed upscaled model has only one coarse degree of freedom (DOF) for each fracture network. Numerical results show that our NLMC method for fractured porous media provides an accurate and efficient upscaled model on the coarse grid.

The paper is organized as follows. In Section 1, we construct an embedded fracture model for poroelastic media. Next, we construct fine grid approximation using FVM for flow problem and FEM for mechanics in Section 2. In Section 3, we construct an upscaled coupled coarse grid poroelasticity model using NLMC method and present numerical results in Section 4.

1. Embedded fracture model for poroelastic medium

The proposed mathematical model of a coupled flow and mechanics in fractured poroelastic medium contains an interacting model for fluid flow in the porous matrix, flow in fracture network and mechanical deformation. The matrix is assumed to be linear elastic and isotropic with no gravity effects. The mechanical and flow models are coupled through hydraulic loading on the fracture walls and using the effective stress concept [35,42]. For fluid flow, we consider a mixed dimensional formulation, where we have a coupled problem for fluid flow in the porous matrix in $\Omega \in \mathcal{R}^d$ $(d = 2, 3)$, and flow in the fracture network on $\gamma \in \mathcal{R}^{d-1}$ (see Fig. 1 for $d = 2$).

Porous matrix flow model. Using the mass conservation and Darcy law in the domain Ω:

$$\frac{\partial m}{\partial t} + \operatorname{div}(\rho q_m) = \rho f_m, \quad q_m = -\frac{k_m}{\nu_f} \operatorname{grad} p_m, \quad x \in \Omega, \tag{1}$$