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Fast and slow strategies in multiplication ☆

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ABSTRACT

In solving multiplication problems, children use both fast retrieval-based processes and slower computational processes. In the current study, we explore the possibility of disentangling these strategies using information contained in the observed response latencies using a method that is applicable in large data sets.

We used a tree-based item response-modeling framework (De Boeck & Partchev, 2012) to investigate whether the proposed qualitative distinctions in fast and slow strategies can be detected. This so-called fast-slow model was applied to responses to a set of multiplication items, totalling more than 180,000 responses, collected in an online computer-adaptive training environment for mathematics.

Parameters describing person characteristics (ability) and item characteristics (easiness) are estimated with the model. Both item and person characteristics differed between fast and slow processes and match predictions from substantive models of multiplication. Moreover, the parameters allowed us to describe the fast and slow strategies in more detail. Results emphasize the utility of the fast-slow model in the detection of strategies in multiplication but also in other areas of cognition and learning where strategies are expected.

1. Introduction

The concept of strategy is central in the study of human problem solving. Important aspects of problem solving behavior such as accuracy, duration, and type of errors, are due to the choice of the solution strategy. For instance, in solving arithmetic items, people may use either retrieval from memory or a computational strategy (Ashcraft & Guillaume, 2009; Dowker, 2005; LeFevre et al., 1996), where the former typically requires less time than the latter. In the case of basic multiplication (for example single-digit problems), detailed models for the retrieval process exist (Geary, Widaman, & Little, 1986; Verguts & Fias, 2005), and several models for computational strategies have been developed as well (Imbo, Vandierendonck, & Rosseel, 2007; Lemaire & Siegler, 1995). These models make different predictions about item difficulty and solution time (van der Ven, Straatemeier, Jansen, Klinkenberg, & van der Maas, 2015).

When measuring arithmetic ability by using psychometric tests, such as in IQ tests, individual differences in strategy choice are usually not taken into account. Arithmetic ability is ultimately tested by counting the number of correct items that participants solve in any particular test (e.g., Aunola, Leskinen, Lerkkanen, & Nurmi, 2004; Liu, Wilson, & Paek, 2008). Different patterns of response times and errors are hence ignored when the aim is to compare individuals on a scale of arithmetic ability. Using the number of correct responses may be warranted when testing and comparing test takers, but may be inappropriate when concerned with studying development and understanding ability differences. In the latter case, different qualitative processes or strategies should be considered.

In spite of the importance of the strategy concept, detecting strategies is still a major challenge in many areas of cognitive science. Verbal reports and neural imaging features are both correlated with strategy choice (Jost, Beinhoff, Hennighausen, & Rösler, 2004; Price, Mazzocco, & Ansari, 2013; Tenison, Fincham, & Anderson, 2014), but both also have pitfalls as strategy indicators. Verbal reporting, the most commonly accepted method of strategy detection, may interfere with the solution process and bias strategy choice (Kirk & Ashcraft, 2001; Reed, Stevenson, Broens-Paffen, Kirschner, & Jolles, 2015). Another important problem with using verbal reports for detecting strategy choice is that it is time-consuming and thus not feasible in combination with large scale automatic assessment of arithmetic abilities, which is very common nowadays. The latter problem also applies when using neural patterns to identify strategy choice. A third approach, whereby strategies are assessed through a combination of latencies and accuracy, is more promising. The utility of response times, obtained with large-scale computer-based assessment, has already been demonstrated for the detection of individual differences in reading literacy and problem

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solving (Goldhammer et al., 2014). For the detection of strategies we use a different approach. This approach will be applied to developmental data on multiplication skills but applications to other cognitive skills are possible as well (see Coomans, Hofman, Brinkhuis, van der Maas, & Maris, 2016), as long as strategies are associated with diverging patterns of accuracy and response times.

An important developmental trend in learning multiplication can be described by changes in strategy choice. Initially children will apply various slower computational strategies (Freudenthal, 1991). Over time, these computations become more sophisticated (Lemaire & Siegler, 1995). Through practicing multiplication, children will build up a network of associations between numbers. When this network is sufficiently strong, children will be able to confidently retrieve answers to items, and will tend to use faster retrieval from this network instead of a slower computational strategy (Siegler, 1988). This development from computation to automaticity is in line with the more general theory on skill acquisition (Ackerman, 1988; Ackerman & Cianciolo, 2000). Children with learning difficulties do not show this typical transition from computational to retrieval strategies (De Smedt, Holloway, & Ansari, 2011; De Visscher & Noël, 2014). After years of practice, adults will rely predominantly on memory retrieval for single digit multiplication (LeFevre et al., 1996). Hence, the largest divide in strategy choice is whether children and adults use a retrieval strategy or a computational strategy.

In this paper, we investigate whether the fast-slow model (DiTrapani, Jeon, De Boeck, & Partchev, 2016; Partchev & De Boeck, 2012) allows for automatic analyses of strategy use. The fast-slow model is based on splitting the data into fast and slow responses and estimating separate parameters for each of the processes. A third process, based on the response latencies, indicates choice for the fast or slow process. This approach is intermediate between the purely psychometric approach of fitting IRT models to capture multiplication ability on a single latent trait (e.g., Aunola et al., 2004; Liu et al., 2008) and the purely cognitive approach of using computational models to predict response accuracy based on problem characteristics and strategies (partial abilities) (e.g., de la Torre & Douglas, 2008).

We will first introduce the fast-slow model, derive predictions for the case of multiplication, and then apply the model to a data set. This data set includes a large set of responses, both accuracy data and response times, collected with a popular Dutch online adaptive learning environment for mathematics; the Math Garden (Klinkenberg, Straatemeier, & van der Maas, 2011; Straatemeier, 2014).

1.1. The fast-slow model

The fast-slow model is a tree-based item response theory (IRT) model (De Boeck & Partchev, 2012). The rationale of this model is that responses are governed by one of two processes, one fast and one slow, that can be separated by an additional observed variable, in this case the (recoded) response times. The response times are recoded to either fast (1) or slow (0), which serves as an approximation of the underlying process and is modeled as a latent speed dimension. This tree model can be formulated as follows, assuming that a (unidimensional) Rasch model (Rasch, 1960) holds in dimension d, where $d = 1,2$, or 3 and denotes the speed-, fast- and slow dimensions, respectively. In these dimensions, the probabilities of respectively a fast response, a fast and correct response, and a slow and correct response are modeled using a Rasch model. In the Rasch model, the probability of a correct (or for the speed dimension a fast) response of a person p on an item i in dimension d is given by the logistic function:

$$P(x_{pid} = 1|\theta_{pd}, \beta_{id}) = \frac{exp(\theta_{pd} + \beta_{id})}{1 + exp(\theta_{pd} + \beta_{id})}, \quad (1)$$

where θ_{pd} denotes the ability of person p and β_{id} denotes the easiness of item i on dimension d. Hence, the full model has three sets of person parameters, and three sets of item parameters: θ_{p1} reflects the overall probability of a person to generate a fast response, θ_{p2} reflects the ability to give a fast and correct response, and θ_{p3} reflects the ability to give a slow and correct response. Likewise, item easiness parameters correspond to the probability that items are answered fast versus slow (β_{i1}, with a high β_{i1} indicating a high probability of a fast response), the probability of a correct response given that the response was fast (β_{i2}), and the probability of a correct response given that the response was slow (β_{i3}).[1] In line with De Boeck (2008), both $\boldsymbol{\theta_p} = (\theta_{p1},\theta_{p2},\theta_{p3})$ and $\boldsymbol{\beta_i} = (\beta_{i1},\beta_{i2},\beta_{i3})$ are treated as random variables with $\boldsymbol{\theta_p} \sim \mathcal{N}(\boldsymbol{\mu_\theta}, \boldsymbol{\Sigma_\theta})$ and $\boldsymbol{\beta_i} \sim \mathcal{N}(\boldsymbol{\mu_\beta}, \boldsymbol{\Sigma_\beta})$, constraining $\boldsymbol{\mu_\theta}$ to zero to identify the model (see Appendix A for a description of the model estimation procedure).

1.2. Qualitative differences in the fast-slow model

Within the fast-slow model, qualitative differences between fast and slow processes would be reflected by a different ordering of the item parameters, person parameters or both, in the fast compared to the slow component of the model. Hence, to test the hypothesis that the fast and slow processes differ qualitatively, the full fast-slow model with different item parameters for the fast and the slow process as well as different person parameters for the two processes is compared against three constrained versions of the model. This resulted in four different models: (1) the full fast-slow model, (2) constrained item parameters: i.e., $\beta_{fast} = \beta_{slow}$, (3) constrained person parameters: i.e., $\theta_{fast} = \theta_{slow}$, and (4) a baseline model in which both item and person parameters are constrained. If one, or both, constraints resulted in a worse model fit (in terms of prediction; see Section 2.2), this would support the notion that indeed qualitatively different processes were involved in the fast and the slow responses. However, from a measurement perspective different item parameters do not necessarily suggest that the person parameters are different, since these abilities could be highly correlated (the same holds for item parameters if person parameters are different).

Whenever a constraint was imposed we allowed for a difference in the overall mean and in the variances of the fast and slow item and/or person parameters. This reflects the idea that only a correlation between the fast and slow parameters that is significantly lower than one truly reflects a qualitatively different process. For example, if fast responses are more often correct than slow responses it does not necessarily suggest that slow and fast responses have distinct response processes. It may be that for slower responses, retrieval is simply more difficult. However, if for some persons or items the slow responses are more often (in)correct than the fast responses, thereby influencing the correlations of these parameters, this would indeed suggest that different response processes are involved.

1.3. Empirical predictions for a fast-slow model of multiplication processes

Given the observed qualitative differences between fast and slow strategies in multiplication (LeFevre et al., 1996), the full fast-slow model is expected to describe the data best. In this model, both item and person parameters have different estimates in the fast compared to the slow process. It is expected that the fast process will more often match fact retrieval and that the slow process will more often match computational strategies. If this is the case, some parameter estimates of the processes should relate differently to item and person characteristics. Finding that these relations match common findings in the multiplication literature would support the claim that the fast-slow model is a useful method to identify strategies in multiplication at the individual level.

[1] For the readability of the remainder of the paper, we refer to θ_{p1}, θ_{p2} and θ_{p3} as θ_{speed}, θ_{fast}, and θ_{slow}, and to β_{i1}, β_{i2} and β_{i3} as β_{speed}, β_{fast}, and β_{slow}.