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Influence of confining pressure-dependent Young's modulus on the convergence of underground excavation

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A B S T R A C T

The actual convergence of an excavation located in fractured rock mass or the soft rock is largely different with the theoretical result in many cases. Experimental results showed that the influence of confining pressure on Young's modulus is very significant. This paper attempted to illustrate the influence of the confining pressure-dependent Young's modulus in the ground reaction analyses of mountain tunnel. Firstly, the relationship between Young's modulus and confining pressure was described as a non-linear function according to the test results. Based on the plane strain axial symmetry assumption and the incremental theory of plasticity, equilibrium equations and compatibility equations of rock mass around a circular tunnel were deduced theoretically. Based on fourth Runge-Kutta method, a semi-analytical solution was achieved. Considering the effect of confining pressure on Young's modulus, the stress and deformation of rock mass around tunnel was calculated by both analytical and numerical methods. The influence of confining pressure-dependent Young's modulus in surrounding rock was estimated quantitatively. Finally, Tawara saka Tunnel in Japan was taken as an example to explain the influence of confining pressure-dependent Young's modulus. The results showed that the error with respect to the monitoring data was largely reduced with the confining pressure-dependent Young's modulus model, which indicated the necessity of considering the non-uniform distribution of Young's modulus.

1. Introduction

In engineering practice, the analytical and numerical methods are inevitably required to estimate the stress and deformation of surrounding rock mass and to help the design of support system (Li et al., 2008; Huang et al., 2015; Zhang et al., 2015; Feng et al., 2017; Wu et al., 2018a). However, the actual convergence of an excavation located in fractured rock mass or soft rock was largely different with the theoretical results in many cases. Accurate rock parameters and constitutive model are indispensable for the convergence and stability evaluation by analytical and numerical methods. The confining pressure effect on rock strength and Young's modulus should to be considered (Hsieh et al., 2014; Cai et al., 2015).

The influence of confining pressure on rock strength has been studied in depth. Many constitutive models considering confining pressure effect has been established (Fang and Harrison, 2001; Alejano et al., 2009). Cui at al. (2015) conducted an elasto-plastic analysis of a circular opening in rock mass with confining stress-dependent strain-softening behaviour. Moreover, Zhang at al. (2018) obtained an elastoplastic coupling solution of circular openings in strain-softening rock mass considering pressure-dependent effect.

The non-uniform distribution of Young's modulus is considered as another important factor. However, there are relatively few studies about the confining pressure effect on the Young's modulus. Numerous papers were contributed to the determination of Young's modulus of rocks (Palmstrom et al., 2001; Isik et al., 2008; Kodama et al., 2013; Agan et al., 2014; Tinoco et al., 2014). Some of them were focused on the relationship of the Young's modulus and the uniaxial compressive strength (UCS), rock mass rating (RMR) and geological strength index (GSI) for different type of rocks (Leite et al., 2001; Gokceoglua et al., 2003; Kayabasi et al., 2003; Karakus et al., 2005; Hoek et al., 2006; Feng et al., 2014).

Some other works were concentrated on the laboratory experiment to verify the confining pressure effect on the Young's modulus, and some empirical equations were obtained (You et al., 2003; Arslan et al., 2008; Wang et al., 2009; Cai et al., 2015; Yang et al., 2016). However, the attempt to describe the exact influence of the confining pressure-dependent Young's modulus of rock mass in the ground reaction analyses is quite few.

Brown et al. (1989) presented a stress-dependent elastic moduli and

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obtained the stresses around the axisymmetric boreholes. It was the earliest literature proposing exponential function of pressure-dependent young's modulus. Nawrocki et al. (1995) studied the damaged zones around openings using radius-dependent Young's modulus by numerical simulation. Zhang et al (2012) obtained a closed-form solution for circular openings modeled by the Unified Strength Theory and radius-dependent Young's modulus. This contribution was the first step in obtaining an analytical solution that considers the effect of confining pressure on the Young's modulus. While, the modulus was defined as a direct function of radius rather than the confining pressure, which was not exactly conform to the actual behaviour of surrounding rock mass. Therefore, more work need to be done to get an analytical solution for circular openings considering confining pressure-dependent Young's modulus.

In engineering practice, the distribution of confining pressure is very complex (Jiang et al., 2001). For a general excavation, the confining pressure (minor principal stress) acting on the excavation surface is zero. It increases gradually with the increasing distance between the element and the excavation surface, and will reach a constant value at locations far away from the excavation (Carranza et al., 1999; Hasanpour et al., 2015). Hence, it is necessary to consider the stress field change in the rock mass surrounding the excavation to accurately predict the ground response, especially in deep buried excavations.

Considering the effect of the confining pressure on Young's modulus, the stress and deformation of rock mass around a circular tunnel were calculated by both analytical and numerical methods. The influence of the confining pressure-dependent Young's modulus in surrounding rock was estimated quantitatively in the ground reaction analyses.

2. Relation of confining pressure and Young's modulus

Generally, the Young's modulus of rock mass was often assumed to be uniform in the ground reaction analyses (Graziani et al., 2005; Hasanpour et al., 2015). However, it was observed that the Young's modulus around an excavation was not constant, but rather non-uniform (Zhang et al., 2012; Cai et al., 2015). The Young's modulus of rock mass depends on many factors such as rock quality and confinement. In particular, confinement has a large influence on the Young's modulus. Hence, the stress redistribution due to excavation has a profound influence on the Young's modulus in underground engineering.

Based on a large amount of laboratory test results, You et al., (2003) pointed out that high confining pressure influence the Young's modulus of specimen from weathered rock or weak rock significantly. The relationship between confining stress and Young's modulus for rock masses was approximate to be exponent dependence. The increasing of fiction in the fissures with confining pressure reduces the shear slide, which makes Young's modulus higher.

To obtain a general function to describe the non-linear Young's modulus model, Cai et al. selected four sets experimental data of different rocks (Meglis et al.,1996; He et al., 2006; Mohammad et al., 2013; Cai et al., 2015), and get the best-fit curves. Fig. 1 presented the relationship between the confining pressure and the Young's modulus for the selected test data and the best-fit curves using the non-linear weighted fitting method. The best-fit equations that correspond to different rocks were also shown in Fig. 1.

Based on the fitting results, the non-linear model of Young's modulus and minor principal stress was shown in Fig. 2. A non-linear function was proposed to describe the relationship between the Young's modulus E and confining pressure σ_3 (Cai et al., 2015):

$$E = E_{max} - (E_{max} - E_0)e^{(-a*\sigma_3)} \tag{1}$$

where E_{max} is the maximum Young's modulus at the critical confining pressure, E_0 is the Young's modulus at no confining condition, and a is a model constant. This function can describe the curves very well for rock masses at non-uniform confinement condition. The physical meaning of the properties in Eq. (1) is clear. E_{max} can be considered as the Young's modulus of rock mass at in-situ stress state; E_0 can be viewed as the minimum Young's modulus at the excavation surface, and a controls the non-linearity of the curve and it varies for different rock masses. The influence of model constant a will be discussed later.

As the confining pressure could influence the Young's modulus dramatically, it was necessary to estimate the influence of the non-linear Young's modulus model on the deformation and failure characteristics of rock mass near excavation boundaries. Because of the lacking of well controlled in-situ experiments, field data was rarely available to determine the influence of the non-linear Young's modulus. Fortunately, the development of semi-analytical and numerical methods based on the computer makes it possible to estimate the influence of the confining pressure-dependent Young's modulus in surrounding rock quantitatively in the ground reaction analyses.

3. Ground reaction analyses of a circular tunnel with confining pressure-dependent Young's modulus model

The confining pressure-dependent Young's modulus model was applied in the ground reaction analyses of a circular tunnel to reveal its influence on the tunnel convergence.

3.1. Problem description

The excavation of long deep tunnels with circular cross section under hydrostatic in-situ stress condition could be considered as an axial symmetry plane strain problem, while neglecting the influence of gravity, and restricting the out-of-plane principal stress as intermediate stress (Li et al., 2013; Mohamad et al., 2013). The geomechanics sign convention was employed, and the radial displacement towards tunnel axis was taken as positive consequently. The stress and displacement redistributions (or namely ground responses) after excavation were evaluated with different Young's modulus models.

3.2. Equilibrium equations for rock mass

Consider an infinitesimal volume in the radial direction as shown in Fig. 3. The rock mass is subjected to a radial stress σ_r, a tangential stress σ_t. The static equilibrium condition of the infinitesimal rock mass volume can be formulated as:

$$\sigma_r r d\omega L_z + 2\sigma_t dr L_z \sin\frac{d\omega}{2} = (\sigma_r + d\sigma_r)(r + dr)d\omega L_z. \tag{2}$$

where, r is the radius of the infinitesimal volume, $d\omega$ is the loop angle, dr is the size in the radial direction, L_z is the size in the axial direction of the tunnel. It is clear that the confining pressure is the radial stress for a circular symmetric tunnel. Noticing that $\sin(\frac{d\omega}{2})$ approximately equals $\frac{d\omega}{2}$ since $d\omega$ is an infinitesimal, the equilibrium equation can be deduced as:

$$\frac{d\sigma_r}{dr} = \frac{\sigma_t - \sigma_r}{r}. \tag{3}$$

When applying Eq. (3) to the elastic region, where the stress state of rock mass should verify the hydrostatic in-situ stress condition that the sum of σ_r and σ_t equals $2P_0$, where P_0 is the in-situ stress. The equilibrium equation for elastic region can be formulated as:

$$\frac{d\sigma_r}{dr} = \frac{2P_0 - 2\sigma_r}{r}. \tag{4}$$

When applying it to the plastic region, where the stress state of rock mass should verify the Mohr–Coulomb failure criterion, the equilibrium equation for the plastic region can be formulated as:

$$\frac{d\sigma_r}{dr} = \frac{(K_p - 1)\sigma_r + \sigma_c}{r}. \tag{5}$$

where, K_p is the passive coefficient and remains unchanged within the