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Guided waves propagation in anisotropic hollow cylinders by Legendre polynomial solution based on state-vector formalism

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ABSTRACT

A spectral approach was presented in the computation of dispersion curves for the general anisotropic hollow cylinders. The derivation is based on the hybrid method of the state-vector formalism and Legendre polynomials expansion, which was previously adopted for the anisotropic plates. This method will lead to an eigenvalue/eigenvector problem for the calculation of wavenumbers and displacement profiles. This hybrid method avoids solving the transcendental dispersion equation. A closed-form solution for the hollow cylinder, involving multiple integral expressions, is demonstrated. A stable scheme for the integration expansion was established by re-expanding the expansion operators from the first round Legendre polynomial expansion versus the displacements. Usually, the traditional matrix methods are based on root-finding algorithms, which is difficult to implement in anisotropic tubes. In this research, the hybrid approach we proposed provides a reliable mathematical solution of wave propagations in an anisotropic hollow cylinder. Applications will be illustrated using isotropic and orthotropic hollow cylinders, in which the isotropic case agrees well with the results by global matrix method. The dispersion curves of orthotropic hollow cylinders, when the out radius set to approximate infinity, are compared to its corresponding anisotropic plate, which is obtained from our previous work. Furthermore, the displacement and stress profiles will be given and analyzed for an orthotropic tube, which has 10 mm thickness with an out radius of 50 mm.

1. Introduction

The hollow cylinders are widely used in many engineering applications, such as pipelines of offshore oil platform, pressure pipes, automobile transmission shaft, aerospace structures, etc. Discussions regarding the vibration and wave propagation characteristics in the hollow cylinder were prosperous in recent decades. Elastic guided waves possess great potentials for nondestructive evaluation [1,2] and structural health monitoring [3,4] of the cylindrical structures due to their ability to detect the inaccessible regions. The propagation behaviors of guided waves are required for pipeline inspections and evaluations. It is necessary to determine the dispersion curves for the understanding of wave propagation. The excitation efficiency and mode selection should be the key points in the system design. In addition, the priori knowledge of guided waves can also be used to explain the received wave signals in order to characterize the defects [3]. However, these investigations on wave propagation characteristics require a comprehensive mathematical model, in which the conventional matrix method is difficult to implement in hollow cylindrical anisotropic structures. The analytical solution of isotropic cylindrical structures has been thoroughly investigated by Mindlin [5], Pao [6,7], Gazis [8,9] and Zemanek [10], Viktorov [11], Achenbach [12], and Auld [13]. But it is more complex or even impossible to obtain analytical solutions for the anisotropic pipes. Dispersion software, developed by the NDE group at the Imperial College, clearly pointed out its limitations: when dealing with cylindrical structures, the material must be isotropic [14].

The polynomial series approach has been presented to solve the wave propagation problem for more than fifty years. Mirsky [15], McNiven and Mengi [16] developed Frobenius power series expansion to solve elastic wave propagation in orthotropic cylinders. The displacement components in the circumferential and axial directions are expressed in term of trigonometric functions, while the radial displacement field along the thickness is expanded by power series. But the power series are not orthogonal, and will give rise to numerical

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instability. After that, Maradudin et al. [17] developed the Laguerre orthogonal polynomial approach to solve linear acoustic waves in homogeneous semi-infinite wedges. Later, the Legendre polynomial [18,19] method was extended to investigate the plate and pipe structures with finite-thickness. This method used a series of Legendre polynomials and trigonometric functions to expand the displacement field and then substituted them into the wave equations, in order to transform the wave propagation problem into the eigenvalue problem. The Legendre polynomial method has been applied to examine the problems of wave propagation in the functionally graded plates and pipes [20–22]. Sanderson [23] chose an N^{th} order polynomial to express the displacement field along the thickness direction analytically. Then, the analytic solution of stiffness and mass matrix is deduced to realize the fast calculation of the dispersion curve for isotropic pipes. In addition, the power series expansion method has also been proposed to expand the unknown field to solve the dispersion curve of the Lamb waves in the gradient material plate [24]. Fourier series expansion method can also be used to solve the dispersion curve of the circumferential wave in the transverse isotropic cylindrical pipes [25,26].

Another interesting method is the spectral method, which has also been adopted to solve dispersion curves in the plate and pipes [27–30]. This method used Chebyshev spectral differentiation matrices to establish the differential equation and boundary conditions in the form of a matrix eigenvalue. Spectral methods are particularly well suited to solving eigenvalue problems. This is due to that the number of interpolation points required for a given accuracy is considerably smaller than for finite difference or element method [28]. This spectral approach has been confirmed as a flexible and fast method to calculate the dispersion curves of the plate and cylindrical structures. Karpfinger et al. [27], Zharnikov et al. [28], Yu et al. [29] and Quintanilla et al. [30] calculated the dispersion relation of the guided wave in the plates and pipes, utilizing spectral method.

In order to overcome the limitation of the conventional matrix methods in calculating the dispersion characteristics of anisotropic cylinder structures, we derived the wave motion equation in term of state matrix form in the cylindrical coordinates. Then, the solving process of the hollow cylindrical structure was transformed into a linear eigenvalue problem, following the basic principle of the spectral method. The proposed method is referred to as the state vector and Legendre polynomial hybrid method (SV-LPHM). Recently, We have formulated SV-LPHM in Cartesian coordinates for the general anisotropic plates [31]. In this research, we extend the SV-LPHM into the analysis of the complicated propagation of guided waves in the general anisotropic hollow cylinder. The basic idea of SV-LPHM is that the displacement field components in the hollow cylinder are expanded by the Legendre polynomials. The eigenvalues and eigenvectors are obtained by projecting the governing state matrix equations onto the complete orthogonal basis, consisting of Legendre polynomials. Another feature of this method is that elastic wave equations of the general anisotropic material in the cylindrical coordinate system is expressed in the matrix form by the displacement vector and the stress vector. The state matrix equation is derived mainly by dividing the stress components into three groups [σ_{rr}, $\sigma_{r\theta}$, σ_{rz}], [$\sigma_{\theta r}$, $\sigma_{\theta\theta}$, $\sigma_{\theta z}$] and [σ_{zz}, $\sigma_{z\theta}$, σ_{zz}]. The reason for this procedure is that the stress vector τ_r in the outer and inner surface of the hollow cylinder are [σ_{rr}, $\sigma_{r\theta}$, σ_{rz}]. Noting that, $\boldsymbol{\tau_r}$ needs to be replaced by $\boldsymbol{r\tau_r}$ in the cylindrical coordinate system. This unique arrangement greatly simplifies the formulation and gives a concise form of the matrix equation for $\boldsymbol{u}$ and $r\boldsymbol{\tau_r}$.

When dealing with a general anisotropic hollow cylinder by using state vector formulation, the displacement and stress boundary conditions can be processed in a consistent way. The Legendre polynomials are chosen to ensure that closed-form solution can be found for all involved integral expressions. Few reports provided an analytical solution to the wave propagation problem of a general anisotropic cylindrical structure. The investigations outline that it is advisable to deal with the anisotropic problems in the cylindrical coordinates as well as in the

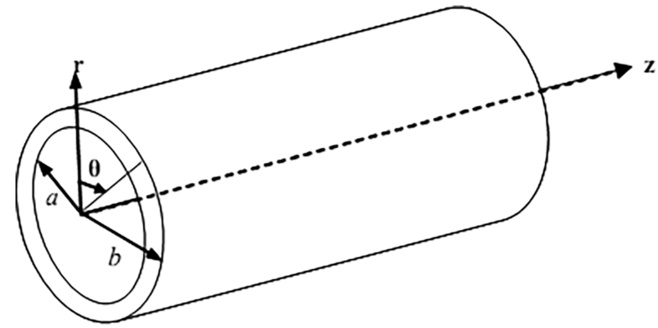

Fig. 1. Spatial coordinate system for the generally anisotropic hollow cylinder.

Cartesian coordinates under the state formalism framework. In Section 2, we will introduce the state vector to describe the governing equations in anisotropic cylindrical geometries and the relevant boundary conditions in term of matrix form. The dispersion equations in term of matrix form are calculated numerically, and the displacement vector is expanded by Legendre polynomials. The most critical step in performing SV-LPHM for the hollow cylindrical structures will be mentioned. In Section 3, we give the numerical results for the isotropic cylinder, in order to highlight its capabilities. Comparison is made with the global matrix method (GMM) method. Section 4 presents the anisotropic cases to validate its effectiveness and stability. Section 5 gives the wave structures and stress profiles of the guided waves in the hollow cylinders. General concluding remarks are added in Section 6.

2. The description of the problem and the basic equations

2.1. The problem definition

Considering a generally anisotropic hollow cylinder that extends infinitely in the z directions, and has an arbitrary thickness H in the r direction (Fig. 1). The longitudinal axis of the anisotropic cylinder is chosen to coincide with the z-direction. The outer and inner radius of the anisotropic cylinder are a and b, respectively. Using r, θ, and z to denote the radial, circumferential and axis of the anisotropic cylinder, respectively, the corresponding displacement components are u_r, u_θ, u_z. We assumed that the hollow cylinder is constructed of a linearly homogeneous elastic material. For an anisotropic material, the stress-strain relation in cylindrical coordinate system is described as follows:

$$\sigma_{ij} = C_{ijkl}\varepsilon_{kl}, \qquad i, j, k, l \in \{r, \theta, z\} \tag{1}$$

where C_{ijkl} and ε_{kl} are represented stiffness tensor and strain tensor in cylinder coordinates, respectively. The strains components in cylinder coordinate system can be expressed in term of the displacements:

$$\begin{aligned}\varepsilon_{rr} &= \frac{\partial u_r}{\partial r}, \quad \varepsilon_{\theta\theta} = \frac{u_r}{r} + \frac{1}{r}\frac{\partial u_\theta}{\partial \theta}, \quad \varepsilon_{zz} = \frac{\partial u_z}{\partial z},\\ \varepsilon_{\theta z} &= \frac{1}{r}\frac{\partial u_z}{\partial \theta} + \frac{\partial u_\theta}{\partial z}, \; \varepsilon_{rz} = \frac{\partial u_z}{\partial r} + \frac{\partial u_r}{\partial z}\\ \varepsilon_{r\theta} &= \frac{1}{r}\frac{\partial u_\theta}{\partial \theta} + \frac{\partial u_\theta}{\partial r} - \frac{u_\theta}{r}\end{aligned} \tag{2}$$

Without considering body force, the equations of equilibrium in cylinder coordinates can be written as:

$$\begin{aligned}&\frac{\partial \sigma_{rr}}{\partial r} + \frac{1}{r}\frac{\partial \sigma_{r\theta}}{\partial \theta} + \frac{\partial \sigma_{rz}}{\partial z} + \frac{\sigma_{rr} - \sigma_{\theta\theta}}{r} = \rho\frac{\partial u_r}{\partial t^2}\\ &\frac{\partial \sigma_{r\theta}}{\partial r} + \frac{1}{r}\frac{\partial \sigma_{\theta\theta}}{\partial \theta} + \frac{\partial \sigma_{\theta z}}{\partial z} + \frac{2\sigma_{r\theta}}{r} = \rho\frac{\partial u_\theta}{\partial t^2}\\ &\frac{\partial \sigma_{rz}}{\partial r} + \frac{1}{r}\frac{\partial \sigma_{\theta z}}{\partial \theta} + \frac{\partial \sigma_{zz}}{\partial z} + \frac{\sigma_{rz}}{r} = \rho\frac{\partial u_\theta}{\partial t^2}\end{aligned} \tag{3}$$

The stress and displacement component in the inner and outer surface of the hollow cylinder are given as:

$$S(r) = \{\sigma_{rr} \quad \sigma_{r\theta} \quad \sigma_{rz}\} \tag{4}$$

$$D(r) = \{u_r \quad u_\theta \quad u_z\} \tag{5}$$