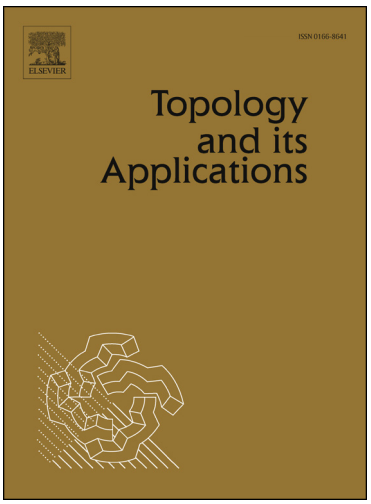

Representing topologies using partially ordered semigroups

Ralph Kopperman, Homeira Pajoohesh

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REPRESENTING TOPOLOGIES USING PARTIALLY ORDERED SEMIGROUPS

RALPH KOPPERMAN AND HOMEIRA PAJOOHESH

Abstract. Two common ways of using the partially ordered semigroup structure of the reals to model topological spaces are:

Defining a distance into $\mathbb{R}$ and using the balls of positive radius about a point as its basic neighborhoods, and

Seeing if the space is a subspace of $\mathbb{R}$ with its interval topology or its upper toplogy.

We show that many po-semigroups can be used in place of $\mathbb{R}$ and in fact:

Every topological space is induced by a quasimetric into and set of positives in some po-semigroup, and is also a subspace of a po-semigroup with its upper topology.

Also, the following are equivalent for any topological space:

It is completely regular,

It is induced by a pseudometric into and set of positives in some po-semigroup,

It is a subspace of some po-semigroup with set of positives in their induced interval topology.

1. Introduction

We bring the following definitions from [2].

Definition 1.1. A *partially ordered semigroup* (abbreviated to *po-semigroup* below) is a poset $(M, \leq)$ with an associative operation $+$ such that for every $a, x, y \in M$, $x \leq y$ implies $a + x \leq a + y$ and $x + a \leq y + a$. A po-semigroup is called:

abelian if for each $a, b \in M,\ a + b = b + a$,

a $po-monoid$ if it has an *identity*, $0 \in M$ $(a = a + 0 = 0 + a$ for each $a \in M)$,

and *idempotent* if $a + a = a$ for every $a \in M$.

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