Annals of Physics 398 (2018) 146–158

Contents lists available at ScienceDirect

Annals of Physics

journal homepage: www.elsevier.com/locate/aop

Helicity and vortex generation

Albert C.R. Mendes [a], Flavio Takakura [a], Everton M.C. Abreu [b,a,*], Jorge Ananias Neto [a], Patrick Paolo Silva [a], João Vitor Frossad [a]

[a] *Departamento de Física, Universidade Federal de Juiz de Fora, 36036-330, Juiz de Fora, MG, Brazil*
[b] *Grupo de Física Teórica e Física Matemática, Departamento de Física, Universidade Federal Rural do Rio de Janeiro, 23890-971, Seropédica, RJ, Brazil*

ARTICLE INFO

Article history:
Received 18 May 2018
Accepted 17 September 2018

Keywords:
Vorticity
Helicity
Circulation

ABSTRACT

In this work we have considered the generalization of the minimal coupling prescription. We have obtained the Navier–Stokes equation for the charged fluid embedded into an electromagnetic field. We have analyzed the wave feature of the fields. Wave equations for velocity and vorticity were obtained and gauge choices were discussed. We have studied the evolution of helicity and circulation from the Maxwell-type formulation for compressive fluid equations, charged in interaction with an electromagnetic field. We see that for both helicity and circulation there are terms that, in principle, can be considered as source terms or creation of circulation in the dynamics of fluids.

1. Introduction

It has been investigated through the last few years an alternative manner to describe fluid dynamics, which is still an open theoretical problem and motivates this work. Some of us have investigated recently with this issue, already attacked in the literature, concerning compressible fluids [1] and plasma concepts [2], both of them by other authors, of course. In [2], the authors have discussed that the better way to attack the problem would be to reformulate the equations of motion (EOM). The result was a set of Maxwell-type equations to describe the fluid. It is direct to realize that this procedure resulted in a modified EOM structure and consequently we have obtained a generalized

* Corresponding author at: Departamento de Física, Universidade Federal de Juiz de Fora, 36036-330, Juiz de Fora, MG, Brazil.
E-mail addresses: albert@fisica.ufjf.br (A.C.R. Mendes), takakura@fisica.ufjf.br (F. Takakura), evertonabreu@ufrrj.br (E.M.C. Abreu), jorge@fisica.ufjf.br (J.A. Neto), patrick@fisica.ufjf.br (P.P. Silva), jvitor@fisica.ufjf.br (J.V. Frossad).

https://doi.org/10.1016/j.aop.2018.09.008

concept of charge and current associated with the dynamics of the fluid [3,4]. To choose the objects that will compose the main part of this new construction one relied upon the above mentioned transformation of the structure of the EOM. In [5], Lighthill has considered the sound emitted by a fluid flow and the applied stress tensor was deemed as the source of the radiation field.

Besides, analyzing the plasma's EOM in [2], R. J. Thompson has introduced a variation of this new design for these very EOM. The obvious interest is to fathom the thermodynamical concepts to realize the dependence of the energy density ρ on the temperature T relative to the fluid's equation of state represented by $p = \omega\rho$. Moreover, it is well known that the Stefan–Boltzmann law has important results considering black holes thermodynamics [6], where one of them says that the energy density is inversely proportional to the temperature.

The dark energy concept, responsible for the observed acceleration of the Universe and with a negative pressure, rules out all the other forms of energy. If we consider that the existence of dark energy is real, we have to take into account the effects of the application of the generalized second law [7] or the entropy bound [8]. The phantom fields ($\omega < -1$), having a negative kinetic energy, negative temperature and positive entropy, can modify altogether the evolution of black holes and their association with the generalized second law [9,10].

The purpose of this paper is to analyze the equation that governs the evolution of helicity and also to discuss the mechanism of vortex generation from the rate of variation of the circulation associated with the canonical moment $\oint_\gamma \vec{v} \cdot d^3x$. The origin of the magnetic field as well as the vorticity is one of the problems not solved in theoretical physics.

In this paper we will follow a sequence such that in Section 2, we have provided, based on [1], a pedagogical description of the obtention of the Maxwell-fluid equations formalism. In Section 3, we have considered the calculations concerning he generalization of [11]. We have obtained the Navier–Stokes equation for the charged fluid embedded into an electromagnetic field. In Section 4 we have analyzed the wave feature of the fields. Wave equations for velocity and vorticity were obtained and gauge choices were discussed. In Section 5 we have computed the dynamics of helicity and circulation and we described the physics of the results. We have left the final section for the general considerations and conclusions.

2. Maxwell equations for compressible fluids: a review

To turn this paper self-contained, we will describe here the main steps of [1] concerning both compressible fluids and the plasma concepts mentioned above. We will describe also the motivations and the main points behind the physical consequences of such formalism.

During the last decades, there were a great endeavor in order to obtain a kind of Maxwell type structure of fluid systems, which was even named as a kind of formalism. To cite only a few attempts, in 1993, Troshkin [12], presented an analogy between Maxwell's equations and fluid equations for an incompressible fluid ($\nabla \cdot \vec{v} = 0$), which was the first attempt. In 1998, Marmanis [3] obtained an analogy between the incompressible Navier–Stokes equations and the Maxwell ones in order to apply to turbulence. Some other approaches were proposed by Popov [13], Ambeogar et al. [14] and Arovas and Freire in [15], where they considered the case of a two-dimensional superfluid.

Let us begin [1] by describing the electromagnetism through Maxwell's equations for both electric field $\vec{E}$ and magnetic field $\vec{B}$ which are described by

$$\begin{aligned} \nabla \cdot \vec{E} &= q & \nabla \cdot \vec{B} &= 0 \\ \nabla \times \vec{E} &= -\frac{1}{c}\frac{\partial \vec{B}}{\partial t} & \nabla \times \vec{B} &= \vec{J} + \frac{1}{c}\frac{\partial \vec{E}}{\partial t}\,, \end{aligned} \tag{1}$$

where $q = 4\pi\rho, \vec{J} = (4\pi/c)\vec{j}$, ρ and $\vec{j}$ are respectively, the charge density and the current density vector, and c is the light velocity. It is also well known that the vector fields $\vec{E}$ and $\vec{B}$ are depicted through a vector potential $\vec{A}$ and a scalar potential ϕ, by

$$\vec{E} = \frac{1}{c}\frac{\partial \vec{A}}{\partial t} \qquad \text{and} \qquad \vec{B} = \nabla \times \vec{A}\,. \tag{2}$$