

# Surface modes in photonic crystal waveguides coated with a layer of dispersive left-handed material

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## ABSTRACT

In this work, we study the presence of surface modes in two different photonic crystal waveguides characterized by the presence of layers of a dispersive left-handed material. The periodicity required to have a photonic crystal waveguide is considered in two distinct ways. One of them is composed of two perfect electrical conducting rippled surfaces, and the other consists of two at infinite surfaces that surround a periodic one-dimensional array of circular cylindrical inclusions of a perfect electrical conductor, coated with a layer of left-handed material. To study these systems, integral numerical methods were applied to determine their band structures and the electromagnetic field distribution, under the TE and TM polarizations. Numerical results show the presence of surface modes for several frequencies with certain geometric configurations. One of these frequencies is known in the literature and corresponds to the frequency of a Surface Plasmon-Polariton.

## 1. Introduction

Left-handed materials (LHM) are materials with a subwavelength structure that have some peculiar properties not present in ordinary materials and owe their name to the fact that the light vectors  $\mathbf{E}$ ,  $\mathbf{H}$  and  $\mathbf{k}$  form a left-handed triad for a wave propagating through these media [1]. This property implies that LHM have a negative refractive index in some regions of the electromagnetic (EM) spectrum [2]. The first experiments with LHM were developed for the microwave region of the EM spectrum, however, recent results indicate that LHM are now available in the visible and infrared [3–5].

In the spectral regions where LHM present a negative refractive index, some of the classical optical phenomena present an atypical behavior, which makes them potentially useful for new technological applications, such as the reconstruction of subwavelength images [6,7], wave guiding [8], optical sensing [9–11], optical cloaking [12,13], micro strip patch antennas [14], and wave absorbers [15]. As a consequence, LHM had become regular components of a wide variety of systems studied by the scientific community.

EM surface waves (SW) propagate along a surface and decay exponentially in the normal direction away from the interface between two different media [16]. The necessary condition for their existence is that the media on both sides of the surface have dielectric functions with opposite sign. The most studied case of SW is that of an air-metal

interface where a surface plasma wave (SPW) can be supported by the surface under different geometries [19–21].

Interesting novel applications, in data storage [17], superlens effect [18] or biosensing [19], involve the study of SW, and in all these systems it is worth emphasizing that SW at metal-LHM interfaces are of great importance, since they appear in both polarizations TM or TE, while SPW can be only excited under TM polarization. It is also known that SW can appear at LHM-vacuum interfaces [22].

Photonic crystals constitute periodic arrays of different materials with a unit cell of dimensions on the order of the wavelength of light. They are also structured systems that present the potential to develop new technologies in integrated optical circuits [23,24], and sensing devices that could help to the detection of diseases such as diabetes or cancer in their early stages [25–27].

In the modeling and propagation of light through these complex systems, numerical simulations play an important role, since no analytical solutions exist. To date, several modeling methods have been developed considering approximate scalar, or full-vector EM waves, such as the effective index method [28], the plane-wave expansion (PWE) method [29], the localized-function method [30], the multipole method (MM) [31], the beam propagation method (BPM) [32], the finite difference time-domain method (FDTD) [33], the boundary element method (BEM) [34], and the finite-element method (FEM) [35]. Although with these methods it is possible to do different numerical

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calculations, the treatment of complex geometries is an obstacle to obtain accurate results. Integral numerical methods have demonstrated to be powerful in the study of the scattering of EM waves by systems of very complex geometries, such as those including random or fractal surfaces [36].

Although we have published some previous studies in the literature treating SW in photonic crystal waveguides (PCW) that include LHM [37–39], the purpose of present research is to determine the existence of surface modes in similar systems when we use coatings layers instead of bulk LHM. For this purpose, we will consider two physical systems with two-dimensional surfaces: the first system is a waveguide composed of two periodic, perfectly conducting, rippled surfaces. Each of these surfaces has a LHM layer. The second system is a waveguide formed by two perfectly conducting flat surfaces and a periodic array of PEC circular cylindrical inclusions coated with a layer of LHM. As a result, we shall demonstrate the presence of SPW at the vacuum-LHM interfaces of these systems based on the ideas presented in recently-published research [39–41].

This paper is organized as follows. Sec. 2 introduces the numerical integral method for studying the systems considered in this work. In Sec. 3 we obtain photonic band structures and analyze the presence of EM SW for the proposed systems under TM, and TE polarizations. Finally, we present our main conclusions in Sec. 4.

## 2. Theoretical approach

The two different systems considered in this work are composed of PEC, LHM and vacuum media. Assuming that we have a sinusoidal time dependence for the electromagnetic fields  $\Psi_j(\mathbf{r}, t) = e^{-i\omega t} \Psi_j(\mathbf{r})$ , it is well-known that the field function satisfies the two-dimensional Helmholtz equation,

$$\nabla^2 \Psi_j(\mathbf{r}) + \left( n(\omega) \frac{\omega}{c} \right)^2 \Psi_j(\mathbf{r}) = 0, \quad (1)$$

where  $\Psi_j(\mathbf{r})$  represents the electric field  $E_x$  in the case of TE-polarization or the magnetic field  $B_x$  for TM-polarization, in the  $j$ -th medium and  $\mathbf{r} = x\hat{i} + y\hat{j}$  is the position vector in the  $x - y$  plane. In the case of the LHM, the refractive index given by  $n(\omega) = -\sqrt{\mu(\omega)\epsilon(\omega)}$  involves the material properties which are expressed in terms of the magnetic permeability  $\mu(\omega)$  and the electric permittivity  $\epsilon(\omega)$ , both of these functions depending on the frequency  $\omega$ . The speed of light is indicated by  $c$ .

The optical properties of LHM given by  $\epsilon(\omega)$  and  $\mu(\omega)$  are expressed in the form [42].

$$\epsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2} \quad (2)$$

and

$$\mu(\omega) = 1 - \frac{F\omega^2}{\omega^2 - \omega_0^2}, \quad (3)$$

being  $\omega_p$  and  $\omega_0$  the plasma and resonance frequencies, respectively. Taking into account the parameters  $\omega_p = 1.5915$ ,  $\omega_0 = 0.6366$  and  $F = 0.56$  [43,44], the region where the LHM has a negative refractive index lies in the frequency range  $\omega_0 < \omega < \omega_{LM}$  with  $\omega_{LM} = \omega_0/\sqrt{1-F} = 0.9597$ .

### 2.1. PCW composed of two periodic, perfectly conducting, rippled surfaces coated with a layer of LHM

A schematic description of the first system to consider is shown in Fig. 1. In this figure the periodic profiles of the surfaces have period  $P$ , the average width of the waveguide is given by  $b$ , the thickness of the LHM's layer is  $T$  and the surface profiles can be represented by the harmonic functions  $y_1(x) = b + A \cos(2\pi x/P)$  (upper profile) and  $y_2(x) = A \cos(2\pi x/P - \Delta\phi)$  (lower profile), where  $A$  represents the amplitude and  $\Delta\phi$  stands for a phase difference between the two

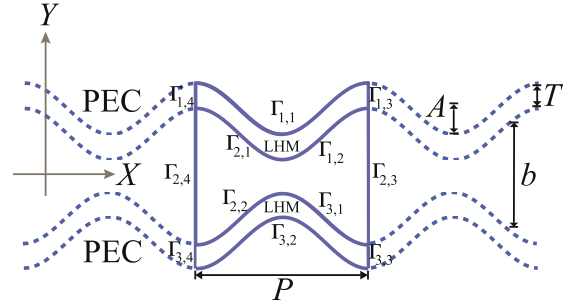


Fig. 1. Graphic description of the photonic crystal waveguides formed by two rippled PEC surfaces coated with a layer of LHM. The  $\Gamma_{i,j}$  contours define the unit cells of the system with periodicity in the  $x$ -direction.

profiles. Each closed contour,  $C_j$ , is formed by four curves  $\Gamma_{j,l}$  with  $l = 1, 2, 3, 4$ , and the corresponding enclosed region. The set of the three regions can be considered as a unit cell of the system. In Fig. 1, it can be seen that the subscript  $j$  refers to the  $j$ -th medium while the subscript  $l$  refers to the  $l$ -th profile in the  $j$ -th medium. The first region ( $j = 1$ ) is a LHM thin layer; the second region ( $j = 2$ ) is vacuum; the third region ( $j = 3$ ) is another LHM thin layer; and finally, up of the curve  $\Gamma_{1,1}$  and down of the curve  $\Gamma_{3,2}$  the regions are PEC media. The set of an infinite number of unit cells is a waveguide of infinite length represented by a perfect crystal. The curves  $\Gamma_{1,2}$  and  $\Gamma_{2,1}$  have associated normal vectors that are in different directions. That is while  $\Gamma_{1,2}$  has a unit normal vector directed outside the first region, the curve  $\Gamma_{2,1}$  has a normal vector directed outside the region with  $j = 2$ .

The periodicity in the  $x$ -direction states a symmetry condition that allows to apply the Bloch's theorem,

$$\Psi(x - P, y) = \exp(-ikP)\Psi(x, y), \quad (4)$$

where  $k$  is the one-dimensional Bloch's vector.

The Green's function associated to the Helmholtz equation for two-dimensional systems is given by

$$G_j(\mathbf{r}, \mathbf{r}') = i\pi H_0^{(1)}(k_j |\mathbf{r} - \mathbf{r}'|), \quad (5)$$

where  $H_0^{(1)}(z)$  is the Hankel's function of the first kind and zero order. Considering the geometry of the unit cell shown in Fig. 1 and applying the Green's second identity for the functions  $\Psi_j$  and  $G_j$ , we obtain

$$\frac{1}{4\pi} \oint_{C_j} \left[ G_j(\mathbf{r}, \mathbf{r}') \frac{\partial \Psi_j(\mathbf{r}')}{\partial n'} - \Psi_j(\mathbf{r}') \frac{\partial G_j(\mathbf{r}, \mathbf{r}')}{\partial n'} \right] ds' = \Psi_j(\mathbf{r})\theta(\mathbf{r}), \quad (6)$$

being  $\theta(\mathbf{r}) = 1$  if  $\mathbf{r}$  is inside the  $j$ -th region and  $\theta(\mathbf{r}) = 0$  otherwise.  $ds$  is the differential arc length,  $\hat{\mathbf{n}}$  is the outward normal vector to the closed curve  $C_j$ , and the observation point  $\mathbf{r}$  is infinitesimally separated of such curve outer to the unit cell. The geometry of the problem is described by representing the points along the contour  $C_j$  with Cartesian coordinates  $X(s), Y(s)$  as parametric functions of the arc-length  $s$  and their derivatives  $X'(s), Y'(s), X''(s)$  and  $Y''(s)$ , up to second order.

In order to solve Eq. (6) numerically, we take a sampling  $X_{n(j,l)} = X_{(j,l)}(s_n), Y_{n(j,l)} = Y_{(j,l)}(s_n)$  along the profiles considered  $\Gamma_{j,l}$  with the corresponding numerical representations of the source functions,  $\Psi_{n(j,l)} = \Psi(X, Y)|_{X=X_{n(j,l)}, Y=Y_{n(j,l)}}$  and the normal derivatives  $\Phi_{n(j,l)} = \partial \Psi(X, Y)/\partial n|_{X=X_{n(j,l)}, Y=Y_{n(j,l)}}$ . In these expressions, the index  $n$  is related to the  $n$ -th point of the point set associated with the profiles  $\Gamma_{j,l}$ . The number of points along the curves  $\Gamma_{j,l}$  are  $N_{j,l}$  with  $N = \sum_{j=1}^3 \sum_{l=1}^4 N_{j,l}$  the total number of points considered. It is important to say that the points  $(X_{n(j,3)}, Y_{n(j,3)})$  on the curves  $\Gamma_{j,3}$  must correspond to those on  $(X_{n(j,4)}, Y_{n(j,4)})$  on the curves  $\Gamma_{j,4}$ , such that  $(X_{n(j,4)}, Y_{n(j,4)}) = (X_{n(j,3)} - P, Y_{n(j,3)})$  and it is necessary to take  $N_{j,3} = N_{j,4}$ . We choose these numbers, so that  $N_x \equiv N_{1,1} = N_{1,2} = N_{2,1} = N_{2,2} = N_{3,1} = N_{3,2}; \quad N_{y1} \equiv N_{1,3} = N_{1,4}; \quad N_{y2} \equiv N_{2,3} = N_{2,4}; \quad N_{y3} \equiv N_{3,3} = N_{3,4}$ . Because the profiles  $\Gamma_{j,3}$  and  $\Gamma_{j,4}$

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