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Measurement of the elastic modulus of nanowires based on resonant frequency and boundary condition effects

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ABSTRACT

The elastic modulus of nanowires can be measured by resonant frequency method, in which the elastic modulus is determined using the classical theory of beam deflection. However, the measurement accuracy of the elastic modulus obtained from the resonant frequency can be largely affected by theoretical models and boundary conditions. In the present paper, a theoretical model is developed by considering nonlocal interactions and boundary condition effects of cantilever support. Based on the proposed model, we investigate the influence of nonlocal parameter and elastic foundation on the elastic modulus of carbon nanotubes (CNTs). The results show that the resonant frequency of CNTs is affected by the clamped cantilever condition. Support stiffness and clamped length of CNTS cannot be ignored in determining the elastic modulus of CNTs, especially for CNTs with low stiffness support. After considering the boundary conditions, the influence of cantilever support on the frequency is effectively removed, and the elastic modulus can be precisely determined by measuring the resonant frequency.

1. Introduction

One-dimensional materials, such as carbon nanotube (CNTs), silicon carbide nanowires have attracted intensive attention in recent years due to their low dimensional and outstanding performances in many applications [1–3]. As one of the most interesting nanomaterials, CNTs have received significant interest because of their extraordinary mechanical and physical properties [4–6]. It is very important to precisely characterize the mechanical properties of CNTs, in particular when CNTs are designed for potential application of nano-electro-mechanical systems (NEMS) [7–10]. The measuring the elastic modulus of CNTs could rely on indirectly method such as thermal vibration amplitudes by Poncharal et al. [11]. The elastic moduli of CNTs were determined when resonantly excited at the fundamental frequency and high harmonics. They studied static and dynamic mechanical deflections of cantilevered CNTs by electrically induced method. The developed method could measure both elastic modulus of CNTs and nano-masses attached to the CNTs. Chen et al. [12] measured the mechanical resonance of microscale quartz fibers to propose a method of obtaining the elastic modulus of nanowires from their fundamental frequency. At present, atomic force microscopy (AFM) is the most promising direct method for analyzing the surface structure. A resonant contact AFM technique was also developed and applied to quantitatively measure the elastic modulus of polymer nanotubes [13,14]. The developments of AFM provide a powerful means for vibrational characterization of materials in the nanoscale. The results showed that the elastic modulus of nanowires or CNTs can be determined by measuring the resonance frequency. However, due to geometrical and clamping constraints, the cantilever support stiffness of stretched CNTs influences directly the measurement accuracy of the vibration frequency.

Since the elastic modulus of CNTs is indirect determined by measuring the resonant frequency, the measurement error depends largely on the corresponding relationship between the elastic modulus and the resonant frequency of CNTs. Thus, it is very important to correctly and effectively determine the elastic modulus dependence with frequency. The mechanical vibrations of CNTs are of importance in various applications, such as nano-mass sensors, high frequency oscillators and other nano-scale devices [15–19]. The vibration property of CNTs was reported on several methods, such as molecular dynamics (MD) [20], molecular mechanics model [21,22], finite element method (FEM) [23,24], and continuum elastic theory [20,25–27]. Li and Chou [28] proposed molecular structural mechanics model to evaluate the flexural frequencies of single-walled CNTs (SWCNTs). In this approach, carbon atoms were considered as concentrated masses placed in bond junctions. Natsuki et al. [26,29] carried out the vibration analysis of SWCNTs and double walled CNTs (DWCNTs), DWCNTs with different

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inner and outer nanotubes [30], and DWCNTs embedded in elastic medium [31], based on the continuum elastic model. When CNTs were embedded in polymer matrices, the effect of surrounding medium on the vibrational property was described by Winkler springs model. Ansari et al. [32–34] studied the oscillatory frequency of ellipsoidal and spherical fullerenes, and carbon nanocones inside CNTs. The oscillatory behavior was investigated based on the continuum approximation along with Lennard- Jones potential energy between carbon molecules.

The essence of the nonlocal elasticity theory [35,36], differing from the classical one in the stress-stain constitutive relations, is that the stress at a reference point is a function of the strain field at every point in the medium. Thus, the nonlocal continuum theory contains information about long range forces between atoms. At small length scales, the lattice spacing between individual atoms for nanostructured materials becomes more important and its effect can no longer be ignored. The small-scale effect on the vibration frequency of CNTs has been rarely considered in past work. Ece et al. [37] investigated the nonlocal elasticity effect on the vibration of CNTs using the nonlocal Timoshenko-beam theory. The results reported that the nonlocal effects should be considered for higher modes of vibration. Wang and Waradan [38] studied free vibrations of SWNTs and DWCNTs utilizing nonlocal continuum mechanics. They showed that there was good consistency between the results obtained by the nonlocal model and the experimental data. Based on the Eringen's nonlocal elasticity theory, a large amplitude vibration of CNTs was investigated under different supported with axially immovable ends [39].

Although the resonance frequency of CNTs was carried out based on different theories, the effect of the cantilever support on the vibration property has not been investigated systematically so far. In the case of partly clamed CNTs, the rigid support cannot correctly describe a state of clamped CNTs [40,41]. This means that the vibration response is different from the existing theoretical model and results because of different boundary conditions. In the paper, an improved mechanics model and theoretical approach were first established to investigate the boundary condition effects on the resonant frequency. The analytical method can be adopted to evaluate the measurement accuracy of the elastic modulus of CNTs by measuring indirectly the resonant frequency. The proposed continuum approach is valid, and able to easily obtain the exact solution of governing difference equations.

2. Theoretical approaches

2.1. Governing equations

In the analysis model, a CNT was considered as a beam on partly elastic supports. As shown in Fig. 1, the CNT beam was of the length L and diameter D, in which L_1 and L_2 were the clamped and exposed lengths of the CNT beam, respectively. The interaction force (p_w) between the CNT and elastic foundation can be described as a Whitney-Riley model characterized by a spring constant (k_w) relative to the stiffness of elastic medium. It is known that nonlocal elasticity theory has been found to successfully describe mechanical behaviors of nanomaterials. In the classical elasticity theory, the stress tensor depends linearly on the strain tensor at a given position, which cannot predict the nonlocal effects. The essence of nonlocal elasticity theory is that the stress field at a reference position depends not only on strain at that position but also on strains at all other points in the domain. The scale effects are accounted in this theory by considering internal size as material parameters. Thus, the nonlocal elastic theory can present the more reliable simulation. According to the nonlocal Euler-Bernoulli beam model [42], the governing differential equation for the transverse vibrations of CNTs is given by

$$EI\frac{\partial^4 w_1}{\partial x^4} + \rho A\frac{\partial^2 w_1}{\partial t^2} - (e_0a)^2\left(\rho A\frac{\partial^4 w_1}{\partial x^2\partial t^2} - \frac{\partial^2 p_w}{\partial x^2}\right) = p_w \quad 0 \le x \le L_1 \tag{1}$$

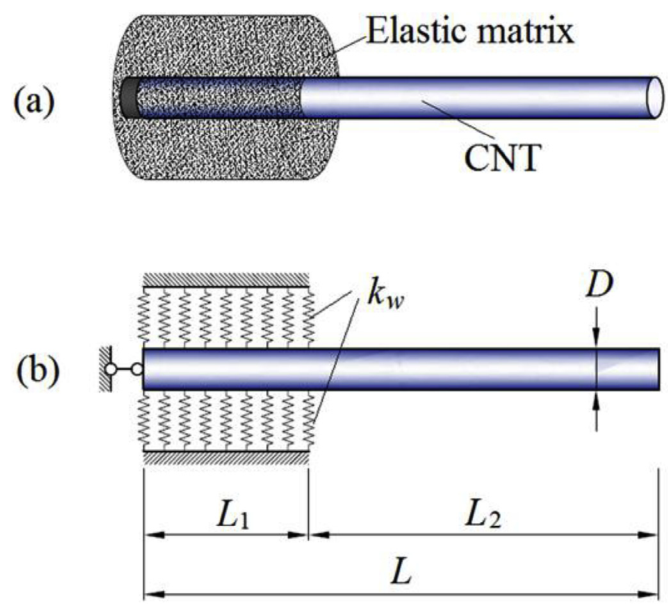

Fig. 1. Schematic illustration of clamped CNT beam embedded in an elastic foundation.

$$EI\frac{\partial^4 w_2}{\partial x^4} + \rho A\frac{\partial^2 w_2}{\partial t^2} - (e_0a)^2\left(\rho A\frac{\partial^4 w_2}{\partial x^2\partial t^2}\right) = 0 \quad L_1 \le x \le L \tag{2}$$

where x and t are the axial coordinate and time, respectively. $w(x, \ t)$ is the deflection of CNT beam, in which w_1 is the clamping part, and w_2 is the exposed part of nanotube length. E is the elastic modulus, I is the moment of inertia and ρ is the mass density. e_0a is the nonlocal parameter, including that e_0 is appropriate to the material, and a is the characteristic internal length of a C—C bond (0.142 nm). p_w is the distributed transverse pressure due to clamping force acting on the CNT. The nonlocal parameter of CNTs is generally taken in the range of 0–2 nm [43]. The interaction force between the CNT and the clamping support can be described as a Whitney-Riley model characterized by a stiffness constant k_w relative to the stiffness of elastic foundation materials, given as

$$p_w = -k_w w_1 \tag{3}$$

2.2. Solution of the governing equations

We consider harmonic vibration of CNTs with angular frequency ω, and $W_j(x), \ j = 1, \ 2$ are the vibration amplitudes of displacement. Thus, the vibrational solution of the differential equations given in Eqs. (1) and (2) can be expressed

$$w_j = W_j(x)\ e^{-i\omega t}, \quad j = 1, \ 2 \tag{4}$$

where ω is the vibration frequency of CNT beam.

Substituting Eqs. (3) and (4) into Eqs. (1) and (2), the following two forms of equations can be deduced

$$EIW_1^{(4)} + (e_0a)^2\gamma W_1'' - \gamma W_1 = 0 \text{ and } \gamma = \rho A\omega^2 - k_w \tag{5}$$

$$EIW_2^{(4)} + \rho A(e_0a)^2\omega^2 W_2'' - \rho A\omega^2 W_2 = 0 \tag{6}$$

General solutions of differential equations Eqs. (5) and (6) can be given as follows:

For $0 \le x \le L_1$, and if $\gamma \le -\frac{4EI}{(e_0a)^4}$,

$$W_1(x) = A_1\cosh\alpha_1 x + A_2\sinh\alpha_1 x + A_3\cosh\alpha_2 x + A_4\sinh\alpha_2 x \tag{7}$$

where