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On the global polynomial stabilization of nonlinear dynamical systems

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A B S T R A C T

This paper presents new criteria for the rational stability of dynamical systems. We introduce basic and useful results on how to construct control laws to assure rational stability. Both Coron and Sontag examples were re-visited. A particular attention is paid to time-varying systems where sufficient Lyapunov conditions are developed guaranteeing the rational stability of this class of systems. As application, the example of double integrator is treated where new time-varying feedbacks rationally stabilizing this system are reconstructed.

1. Introduction

The problem of rational stability and stabilization of dynamical system has been renewed during the last years and becomes a challenging topic in control theory; see, e.g. [1–8]. More precisely, this stabilization means the construction of regular (e.g. Hölderian) feedback laws $u(x)$ that make solutions $x(t)$ of the closed loop system $\dot{x} = f(x, u(x))$ decreasing like c/t^{α} where $\alpha > 0$ measures the velocity of convergence of solutions. This stability appears as a natural solution when the exponential stability is not possible (i.e. the linearization matrix around the trivial solution of $\dot{x} = f(x, u(x))$ is not Hurwitz (e.g. $\dot{x} = f(x, u(x)) = -x^3$). Based on Lyapunov theory, some results on rational stabilizability of nonlinear control systems are developed in [6], and generalizing previous works presented respectively by Hahn in [7, Theorem 23.1], and by Bacciotti and Rosier in [3]. In particular, some extension of known results in stability theory, as well as converse Lyapunov function and backstepping techniques are extended to this type of stabilization.

Basically, the asymptotic stability in Lyapunov sense is popular and known by the community of control theory; see for example, [9–11,3,12,13] and references therein. So, in practical point of view, and in particular for physical systems it seems important to know how solutions of dynamical systems converge to the

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equilibrium point. In addition, for many smooth systems (e.g. Lipschitz systems that may not be linearizable at the origin), the exponential stability [14] and finite-time stability [15] are not always possible. Hence, the rational stability of dynamical systems is a rigorous response to this problematic, and the advantages of this stability theory are remarked in the papers [2,6,8], and showed how the solution of the system can reach the equilibrium point. Among the advantages, that this rational stability is used to overcome the famous Brockett's necessary condition [16] for several systems as well as the Brockett's integrator, chained systems, ship and satellite systems [6,5], by showing that we can stabilize rationally $(n-1)$ components of such system while only one component converges.

In this paper we continue to present new criteria for the rational stability of dynamical systems when some of them can be used for time-varying systems. We give Lyapunov like-sufficient conditions proving that the norm of each solution lies in appropriate domain delimited by two set of curves of the form $|x|t^{\alpha}=c$. This allows to choose the parameter α to accelerate the velocity convergence of the solution to reach rapidly the equilibrium point. The academic examples of Coron and Sontag are treated where regular feedback laws are constructed stabilizing rationally in partial sense booth systems.

The third application is dedicated to double integrator. Indeed; the double integrator or oscillator harmonic is a key system that appears in several mechanical systems. In addition, frequently design approach developed for the double integrator can be easily extended to more general situations by using smooth or non smooth analysis. For these reasons, a lot of attention is paid for the construction of various stabilizing feedback laws for this simple system [17–24,6]. In our work, we have reconstructed regular time varying feedback laws making the rational stability of double integrator, this construction allows us to stabilize partially and polynomially the physical system describing the underwater vehicle.

The paper is structured as follows. The next Section deals with some definitions and notations. The Section 3 deals with the main results where some sufficient conditions are developed. In Section 4, we present several applications where regular feedbacks are build stabilizing partially or completely in polynomial sense some academic examples. The Conclusion is the subject of Section 5.

2. Notations and definitions

In this paper, we adopt the notations: $|.|$ denotes the Euclidean norm on $\mathbb{R}^n$, $\mathrm{L}^1[0,+\infty)$ is the Lebesgue space , $'$ is the symbol of transposition, and “sgn” is the sign function with $sgn(0)=0$, and

$$\mathbb{Q}^{+}_{odd}=\{r\in\mathbb{Q}_{+}: r=\frac{p}{q}, \text{ where } p \text{ and } q \text{ are odd nonnegative integers}\}.$$

For a continuous function $\varphi:[0,+\infty)\to\mathbb{R}$, we denote by $D^{+}\varphi(t):=\limsup_{h\to 0^{+}}\frac{\varphi(t+h)-\varphi(t)}{h}$ the Dini derivative of φ.

It is known from [25, Appendix I, pp 345] that if $\varphi:[0,+\infty)\to\mathbb{R}$, is continuous, then φ is decreasing on $[0,+\infty)$ if and only if $D^{+}\varphi\leqslant 0$.

In addition, the time derivative of a Lyapunov function V along the continuous dynamical system $\dot{x}=f(x), x(0)=x_0$ is defined for all x_0 in Dini sense by [3]

$$D^{+}V(x_0):=\lim\ sup_{t\to 0^{+}}\frac{V(x(t))-V(x_0)}{t}.$$

Results on stability and asymptotic stability in Lyapunov sense with non smooth (only continuous function) Lyapunov function can be found in [25].

We start by recalling the notions of partial rational stability and partial rational stabilizability.

Let the dynamical systems in finite dimension be in the following form

$$\dot{x}_1=X_1(x_1,\,x_2),\ \dot{x}_2=X_2(x_1,\,x_2), \tag{1}$$

where $X=(X_1,\,X_2)$ is a continuous vector field defined on $\mathbb{R}^p\times\ \mathbb{R}^{n-p}$, the state is given by $x:=(x_1,\,x_2)\in\mathbb{R}^p\times\mathbb{R}^{n-p}$ with p an integer such that $0<p\leqslant n$.