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The nonlinear elasticity of hyperelastic models for stretch-dominated cellular structures

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ABSTRACT

For stretch-dominated cellular structures with arbitrarily oriented cell walls made from a homogeneous isotropic hyperelastic material, recently, continuum isotropic hyperelastic models were constructed analytically, at a mesoscopic level, from the microstructural architecture and the material properties at the cell level. Here, the nonlinear elastic properties of these models for structures with neo-Hookean cell components are derived explicitly from the strain-energy function and the finite deformation of the cell walls. First, the nonlinear shear modulus is calculated under simple shear superposed on finite uniaxial stretch. Then, the nonlinear Poisson's ratio is computed under uniaxial stretch and the nonlinear stretch modulus is obtained from a universal relation involving the shear modulus as well. The role of the nonlinear shear and stretch moduli is to quantify stiffening or softening in a material under increasing loads. Volume changes are quantified by the nonlinear bulk modulus under hydrostatic pressure. Numerical examples are presented to illustrate the behaviour of the nonlinear elastic parameters under large strains.

1. Introduction

Solid cellular structures are widespread in nature and in an ever increasing number of biomedical and engineering applications [1–9]. For example, engineered tissue scaffolds provide an environment for growth and regeneration of biological cells [10–19], while natural materials generally incorporate several levels of structural hierarchy, which contribute to their macroscopic physical properties [20–24]. From the modelling point of view, a sub-level in the structural hierarchy can be treated either as a substructure with its own geometry, or as a continuum described by a suitable material model. Advancements in manufacturing techniques is also enabling the creation of new types of materials with several nested hierarchical levels [25,26,24,27]. Such structures promise to explore uncharted territory in materials research [21,4,23], while the recursive nature of their hierarchies brings up questions about self-similar and fractal behaviours [28,25,29,30,22].

When studying cellular structures, the common assumption is that cell walls are linearly elastic with a geometrically nonlinear behaviour. In this case, if the cell walls bend, then the elastic response can be determined from the linear-elastic deflection of a beam [4,5]. However, in many cellular structures, when loaded, the cell walls stretch axially rather than bend. The dominant mechanical behaviour is determined by the architecture and depends on whether the cells are open or closed [31,23]. Stretch-dominated cellular structures, such as octet-truss and body-centred cubic geometries, for example (see Fig. 1), have a higher stiffness-to-weight ratio than bending-dominated ones [31,32,26,21,5,33,34,23]. In addition, biological and bio-inspired materials are often nonlinearly elastic under large strains, and a finite elasticity approach is needed to understand them [35–37].

Microstructure-based models for a cellular solid with open cells of isotropic linearly-elastic material were first proposed by Gent & Thomas (1959) [38], where infinitesimal stretches were assumed. In [39], these models were extended to structures with closed cells containing an ideal gas. For these models, effective Young's modulus and Poisson's ratio under infinitesimal deformations were derived explicitly from the constitutive equations [40,41]. For cellular structures of nonlinearly elastic material under finite strain deformations, a phenomenological continuum model was proposed by Blatz & Ko (1962) [42]. This model reduces to the Gent–Thomas model in the small strain limit [43,44]. Later, it was noted in [45] that Hill's energy functional of hyperelasticity [46] can be used to describe the simple special case of structures where the principal stresses are uncoupled, i.e. depend only on the stretch ratio in the corresponding principal direction. These approaches are based on Ogden-type strain-energy functions for compressible materials extending the incompressible strain-energy functions defined in [47].

For stretch-dominated structures with open or closed cells made from nonlinear elastic materials, in [48,49], novel continuum isotropic

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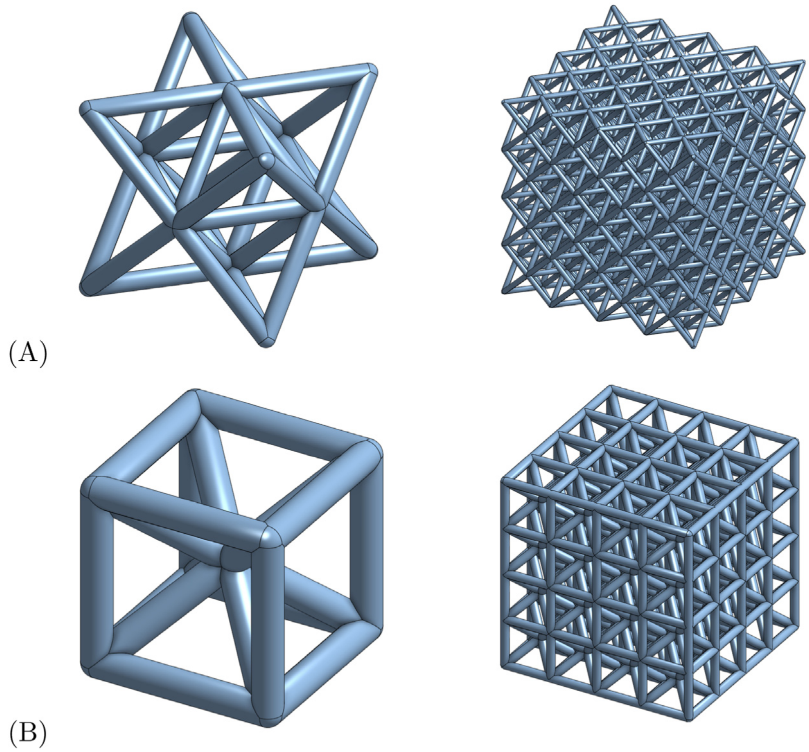

Fig. 1. Examples of stretch-dominated cellular structures: (A) octet-truss and (B) body-centred cubic, at the cell level (left) and at the mesoscopic structural level (right), respectively.

hyperelastic models, at a mesoscopic level, where the number of cells was finite and the size of the structure was comparable to the size of the cells, were constructed analytically from the structural architecture and the material properties at the cell level. For these structures, the cell walls, which were equal in size and arbitrarily oriented, were under finite triaxial deformations, while the joints between adjacent walls were not elastically deformed. The elastic responses at different scales were related by the assumption that, when the structure is subject to a triaxial stretch, each cell wall deforms also by a triaxial stretch, without bending or buckling, and the stretches of the structure and of the cell walls were related by a rotation. Possible instability effects due to cell wall buckling, for example, which could also occur under large deformations, were discussed in [48].

In this paper, we extend the theoretical investigation of the hyperelastic models for structures with neo-Hookean cell components introduced in [48,49], by providing explicit derivations of key nonlinear elastic parameters under large strains, following the formal definition of these parameters given in [50]. In this sense, our explicit multiscale nonlinear elastic analysis and the corresponding numerical illustrations presented here are new. First, the hyperelastic models are summarised in Section 2. Then, for each model, in Section 3, the nonlinear shear modulus is formulated explicitly under simple shear superposed on finite uniaxial stretch. In Section 4, the nonlinear Poisson's ratio is defined under uniaxial stretch and the nonlinear stretch modulus is obtained from a universal relation involving the shear modulus as well. The role of the nonlinear shear and stretch moduli is to quantify stiffening or softening in a material under increasing loads. Volume changes are quantified by the nonlinear bulk modulus under hydrostatic pressure in Section 5. The nonlinear elastic behaviour of the mesoscopic models is illustrated numerically in Section 6 and the numerical results are discussed in Section 7.

2. Hyperelastic models for stretch-dominated cellular structures

In this section, we summarise the general formulation of the continuum hyperelastic models for stretch-dominated cellular structures with open or closed cells proposed in [48,49], and specialise these models to structures with neo-Hookean cell components, which we then analyse in detail in the next sections.

2.1. Geometric assumptions

In open-cell structures, the cell walls consist of the cell edges which form an interconnected network, while in closed-cell structures, the cell walls contain both the cell edges and the cell faces forming disconnected cell compartments. For each structure, all the cell edges are equal and thin, with undeformed thickness t and length L, such that $0 < k = t/L \ll 1$, and meet at joints of approximate thickness t (see Fig. 2, where the joints were slightly enlarged, emphasising that they have non-zero volume).

Open-cell structures. For the open-cell structure, we consider the case where all the cell walls are circular cylinders and the joints are spheres (see Fig. 2A) [48]. Taking the unit volume as the volume of the sphere with radius $R = (L+t)/2 = L(1+k)/2$, which is centred at a joint and contains half of the length of each cell wall connected to that joint (see Fig. 2B), the representative volume fraction of solid material contained in the cell walls, included in this sphere, is

$$\rho_w^{(o)} = \frac{3k^2}{(1+k)^3}. \tag{2.1}$$

Closed-cell structures. For the closed-cell structures, all the cell walls have flat faces and adjacent cell walls meet along cell edges of length L, while adjacent cell edges meet at spherical joints [49]. In this case, setting the unit volume as the volume of a sphere with radius $R = (L+t)/2 = L(1+k)/2$, centred at a joint, the representative volume fraction of solid material contained the cell walls (faces and edges) included in this sphere, is equal to

$$\rho_w^{(c)} = \frac{3k}{(1+k)^2}, \tag{2.2}$$