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Letter to the editor

Comparative study of two order reduction methods for high-dimensional rotor systems

Kuan Lu [a,c,*], Zhenyong Lu [a,b], Yushu Chen [a]

[a] School of Astronautics, Harbin Institute of Technology, Harbin 150001, PR China
[b] School of Management Science and Engineering, Shandong Normal University, Ji'nan 250014, PR China
[c] College of Engineering, The University of Iowa, Iowa City, IA 52242, USA

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ABSTRACT

In this research note, two order reduction methods in the high-dimensional rotor systems are presented: one is the transient proper orthogonal decomposition (T-POD) method and the other is the structure order reduction (SOR) method. The accuracy of the SOR method is verified in comparison to the critical speed error, and the efficacy of the T-POD method is verified via comparing with the nonlinear instability behaviours of the original and reduced models. The SOR method is applied to compare with the T-POD method, and further verifies the efficacy of the T-POD method. The numerical results indicate that both the T-POD method and the SOR method can be applied for order reduction in the high-dimensional rotor system.

1. Introduction

The large rotating machineries are high-dimensional, nonlinear and complex systems, e.g. aircraft engines, gas turbines, etc. The existing nonlinear dynamics theories cannot provide comprehensively qualitative guidance to this kind of systems. So the order reduction methods should be highlighted to study these high-dimensional dynamics problems. The POD method has become one of the most widely used order reduction methods, attracting the attention of researchers in many areas. The POD, also known as Karhunen–Loeve decomposition (KLD), was first presented by Kosambi [1], has been studied by many other researchers, including Jolliffe [2] for the equivalence to principal component analysis (PCA), Berkooz [3] for conceiving in the framework of continuous second-order process, and Liang et al. [4] for the equivalence of the POD, PCA, KLD and singular value decomposition (SVD). More recently, the authors developed the POD method and proposed the T-POD method [5], later extended the T-POD method for the application in rotor system supported by ball bearings [6], and provided the optimal order reduction model based on the physical significance of the T-POD method in Ref. [7], then combined the T-POD method with the singular theory to study the nonlinear instability characteristics of the rotor model with cubic nonlinearity [8].

The rotor models in this note are flexible, and many researchers used the flexible models to study the dynamical characteristics of the rotor systems. For example, Liao et al. [9] carried out numerical efforts by using a reduced-order model with attention to stick–slip interactions between a drill string and an outer shell, Liu et al. [10] discussed nonlinear motions of a flexible rotor with a drill bit, and Vlajic et al. [11] studied torsional vibrations of a Jeffcott rotor subjected to continuous stator contact analytically and numerically for both forward and backward whirling motions.

The motivation of this research note is to compare the results of the original system model with the results obtained by the T-POD [5–8] and SOR [12] method respectively, so that to further verify the efficacy of the T-POD method. The T-POD method from the theory aspect will be verified via comparing with the SOR method which considers the actual system structures. The critical speed, relative error, and nonlinear instability behaviours are studied via comparing the dynamical characteristics of the original systems with the reduced ones. The numerical results of the rotor system indicate that two order reduction methods can provide accurate and efficient estimates of the original system.

2. Brief introduction to the general T-POD method

A brief description of the T-POD method will be introduced in this section based on the author's previous papers [5–8]. The POD method will not be introduced here, for further details see Refs. [7,13].

* Corresponding author at: School of Astronautics, Harbin Institute of Technology, Harbin 150001, PR China.
E-mail address: lukuanyyzb@163.com (K. Lu).

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The T-POD method is proposed based on transient time series; a set of proper orthogonal modes (POMs) are obtained by utilizing POD from the transient response of the original system. The first few order POMs are used to form the projection space and to project original system onto this space. Then we can get the approximately equivalent model of the reduced system.

In general, the multi-degrees of freedom (DOF) system can be written as:

$$\ddot{\mathbf{Z}} = -\mathbf{C}\dot{\mathbf{Z}} - \mathbf{K}\mathbf{Z} + \mathbf{F}\left(\mathbf{Z}, \dot{\mathbf{Z}}, \omega, t\right) \tag{1}$$

where $\mathbf{C}, \mathbf{K}, \mathbf{F}$ are the equivalent matrices of damping, stiffness and force vector. ω is the rotating speed of the rotor.

The displacements information from transient process of various DOFs can be denoted as $\mathbf{z}_1(t), \mathbf{z}_2(t), \ldots, \mathbf{z}_E(t), E$ is the number of DOFs in the original rotor system. Transient time interval displacement sequences of all DOFs are $\widetilde{\mathbf{z}}_i = \left(\mathbf{z}_i(t_1), \ldots, \mathbf{z}_i(t_N)\right)^T, i = 1, \ldots, E$. The number of time intervals is N. We can get the matrix $\mathbf{H} = \left[\widetilde{\mathbf{z}}_1, \ldots, \widetilde{\mathbf{z}}_E\right]$, the dimension is $N \times E$. Then the $E \times E$ self-correlation matrix $\mathbf{J}$ is $\mathbf{H}^{\mathrm{T}}\mathbf{H}$. The eigenvectors of the matrix can be denoted as $\boldsymbol{\varphi}_1, \ldots, \boldsymbol{\varphi}_E$ and the corresponding eigenvalues are $\lambda_1 \geq \cdots \geq \lambda_E$. $\mathbf{L}$ is a matrix formed by the first n orders of $\mathbf{J} = \mathbf{H}^T\mathbf{H}$, and it can also indicate that the matrix $\mathbf{L}$ contains the first n largest eigenvalues of $\mathbf{J}$. The dimension of $\mathbf{L}$ is $E \times n$, take the coordinates transformation on system coordinates $\mathbf{Z}$, then we can get the new coordinates $\mathbf{R}, \mathbf{Z} = \mathbf{L}\mathbf{R}$. Substitute $\mathbf{Z}$ into Eq. (1), we can gain Eq. (2):

$$\mathbf{L}\ddot{\mathbf{R}} = -\mathbf{C}\mathbf{L}\dot{\mathbf{R}} - \mathbf{K}\mathbf{L}\mathbf{R} + \mathbf{F} \tag{2}$$

taking $\left(\mathbf{L}^T\mathbf{L}\right)^{-1}\mathbf{L}^T$ left multiplication on both sides, Eq. (3) can be obtained:

$$\ddot{\mathbf{R}} = -\mathbf{C}_n\dot{\mathbf{R}} - \mathbf{K}_n\mathbf{R} + \mathbf{F}_n \tag{3}$$

where $\mathbf{C}_n = -\left(\mathbf{L}^T\mathbf{L}\right)^{-1}\mathbf{L}^T\mathbf{C}\mathbf{L}, \mathbf{K}_n = -\left(\mathbf{L}^T\mathbf{L}\right)^{-1}\mathbf{L}^T\mathbf{K}\mathbf{L}, \mathbf{F}_n = -\left(\mathbf{L}^T\mathbf{L}\right)^{-1}\mathbf{L}^T\mathbf{F}\mathbf{L}$.

The basis processes of the T-POD method in rotor systems are expressed briefly as follows: (1) extract the transient information of time history response, and the self-correlation matrix can be obtained; (2) the original system is projected to the subspace spanned by the corresponding reduced order modes. The T-POD method is used to study the rotor system model (Fig. 1) in this note. As above, the T-POD method is applied to reduce the original system to the n-DOFs system. n is far less than the DOF number of the original system. The more detailed processes were shown in the author's previous papers [5–8], especially the T-POD method was first proposed and applied in the rotor system model in Ref. [5]. The results in this research note obtained by the T-POD method will be discussed in details in Section 4.

3. The SOR method in rotor model

In this section, the SOR method is outlined, details can be found in [12]. This method is an original method proposed by our research group. Consider a high pressure rotor system depicted in Fig. 1(a), the supporting system is a two-fulcrum structure; the supports are on the front and back fulcrums, 1-0-1supporting scheme. The system contains 11-level discs: 9-level high-pressure compressor, one seal gas disc and one high pressure turbine. The rotor is divided into three portions: the first is 1–3 level discs, cantilever construction; the second is 4–6 level discs; the third is 7–9 level discs and one seal gas disc; the joints are all welded.

The more accurately, complexly discrete dynamics model is established based on structural style and operating principles. A sketch of the dynamical model is shown in Fig. 1(b), the modelling assumptions are as follows:

The 1–9 level discs, seal disc and high pressure turbine disc are equivalent to 11-level hollow discs. They are concentrated at centroid as the lumped mass form, the blade parameters of each level disc are concentrated on the corresponding disc. $R_j, r_j, d_j, m_j, J_{pj}, J_{dj}\,(j = 1, \ldots, 11)$ represent equivalent external radius, internal radius, thickness, mass, axial and diameter moment of inertia of j-level disc. The shaft is simplified as circular hollow beam of uniform cross section, beam-shaft element of Timoshenko. We only consider lateral bending vibration; ignore the effects of torsion and axial force of shaft. The supports are simplified as isotropic elastic support.

The similar model as Fig. 1(b) was used in Ref. [5], the slide bearings were considered to be supported at both ends. The nonlinear dynamic equation can be written as Eq. (4):

$$\mathbf{M}\ddot{\mathbf{Z}} + \mathbf{C}\dot{\mathbf{Z}} + \mathbf{K}\mathbf{Z} = \mathbf{F}\left(\mathbf{Z}, \dot{\mathbf{Z}}, \omega, t\right) \tag{4}$$

$\mathbf{M}, \mathbf{C}, \mathbf{K}$ represent mass, damping and stiffness respectively. $\mathbf{F}$ is the force vector, which includes oil-film force and external excitation. The detailed parameters can be found in section 3.1 of the authors' previous paper [5].

In complex mechanics model, the coordinate and nodes are in Fig. 1(b). The mass and rotational inertial of 1–3 level high pressure discs are concentrated to the 3-level disc, then we get ① level disc which locates at node 2; the 4–9 level and seal gas discs represent ②–⑧ at nodes 3–9; the high pressure turbine disc is simplified as ⑨ at node 10. The discs are connected by hollow shaft sections, the front and back supports are at node 1, 11respectively. The parameters are shown as follows:

$$\begin{aligned} & m_1 = \sum_{j=1}^{3} m_j, m_i = m_{j+2}\,(i = 2, \ldots, 9, j = i) \\ & J_{p1} = \sum_{j=1}^{3} J_{pj}, J_{pi} = J_{pj+2}\,(i = 2, \ldots, 9, j = i) \\ & J_{d1} = \sum_{j=1}^{3} J_{dj}, J_{di} = J_{dj+2}\,(i = 2, \ldots, 9, j = i) \end{aligned} \tag{5}$$

The linear form of dynamical equation of the system is:

$$\begin{aligned} & \mathbf{M}_1\ddot{\mathbf{p}}_1 + \Omega\mathbf{J}_1\dot{\mathbf{p}}_2 + \mathbf{K}_1\mathbf{p}_1 = 0 \\ & \mathbf{M}_1\ddot{\mathbf{p}}_2 + \Omega\mathbf{J}_1\dot{\mathbf{p}}_1 + \mathbf{K}_1\mathbf{p}_2 = 0 \end{aligned} \tag{6}$$

$\mathbf{M}_1, \mathbf{K}_1, \boldsymbol{\Omega}_1$ are mass, stiffness, rotation matrices respectively. Ω is the autorotation angular velocity. The frequency equation is:

$$\left| -\mathbf{M}_1\omega^2 + \mathbf{J}_1\Omega\omega + \mathbf{K}_1 \right| = 0 \tag{7}$$

Based on the formula (7), the critical speed can be calculated when $\Omega = \omega$.

The original model should be reduced based on the SOR theory. In Fig. 2, the model B and C are the reduced models. Here we consider: the bearing position, parameters of shaft sections are unchanged; we only simplify the parameters, number and the position of discs.

In model B, we apply the simplified method on the basis of centroid concentration. Each level pressure disc is concentrated to the position of disc ① centroid, the turbine discs are concentrated to the centroid of the disc ②, the parameters of shaft sections and bearings are invariable. This method guarantees the mass and rotational inertial of discs of reduced model invariable. The parameters of disc ①, ② are $\bar{m}_1 = \sum_{i=1}^{8} m_i, \bar{J}_{p1} = \sum_{i=1}^{8} J_{pi}, \bar{J}_{d1} = \sum_{i=1}^{8} J_{di}$ and $\bar{m}_2 = m_9, \bar{J}_{p2} = J_{p9}, \bar{J}_{d2} = J_{d9}$.

The same method is used in model C, the model is single-disc, the parameters of discs are $\bar{m}_1 = \sum_{i=1}^{9} m_i, \bar{J}_{p1} = \sum_{i=1}^{9} J_{pi}, \bar{J}_{d1} = \sum_{i=1}^{9} J_{di}$. The accuracy of the dual-disc and single-disc models will be discussed in Fig. 5 of Section 4.

4. Results and discussions

In this section, we will study the accuracy and efficacy of T-POD and SOR method. The comparison of the two methods will be discussed, the dynamical equation is shown as Eq. (4) and the detailed parameters of the rotor system are the same as those in the author's paper [5], only the looseness is not considered in this note.