



Low-power peaking-free high-gain observers[☆]

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ABSTRACT

We propose a *peaking-free low-power* high-gain observer that preserves the main feature of standard high-gain observers in terms of arbitrarily fast convergence to zero of the estimation error, while overtaking their main drawbacks, namely the “peaking phenomenon” during the transient and the numerical implementation issue deriving from the high-gain parameter that is powered up to the order of the system. Moreover, the new observer is proved to have superior features in terms of sensitivity of the estimation error to high-frequency measurement noise when compared with standard high-gain observers. The proposed observer structure has a high-gain parameter that is powered just up to two regardless the dimension of the observed system and adopts saturations to prevent the peaking of the estimates during the transient. As for the classical solution, the new observer is robust with respect to uncertainties in the observed system dynamics in the sense that practical estimation in the high-gain parameter can be proved.

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1. Introduction

High-gain observers appeared in the literature at the end of the 1980's and since then have attracted a lot of research attention due to their simplicity and good performance in noise-free settings (see the survey Khalil & Praly, 2014 and references therein). See also their use in the separation principles (Atassi & Khalil, 2000), output feedback stabilization (Teel & Praly, 1994), output regulation (Byrnes & Isidori, 2004) or fault detection (Martinez-Guerra & Mata-Machuca, 2013).

In the design of a “standard” high-gain observer, the high-gain parameter, denoted as ℓ throughout this paper, is usually powered up to n , with n denoting the dimension of the observed state. This fact raises numerical issues in the implementation when the state dimension is high or when the high-gain parameter has to be chosen large to achieve fast estimation. Furthermore, high-gain observers exhibit, during the transient, the so-called peaking phenomenon, namely the state of the observer shows large

peaks of a magnitude that are proportional to ℓ^{n-1} . Last but not least, high-gain observers are known for having high-sensitivity to high-frequency measurement noise, which makes state estimates practically unusable especially when the dimension n is very large. In order to address the peaking phenomenon, different schemes have been proposed in Astolfi and Praly (2017) and Maggiore and Passino (2003). In Astolfi and Praly (2017), the authors modify the observer dynamics under a convexity assumption in order to constrain the state of the observer in some prescribed convex closed set. This technique can be applied to multi-input multi-output nonlinear systems. In Maggiore and Passino (2003), the authors deal with peaking by means of a projection approach. In order to improve the sensitivity to measurement noise, the majority of researchers focused on schemes with time-varying gains, either with switched approaches, Ahrens and Khalil (2009), or with adaptive design, Boizot, Busvelle, and Gauthier (2010) and Sanfelice and Praly (2011). Recently, in Khalil and Priess (2016), a low-pass filter has been proposed in order to reduce the effect of measurement noise in output feedback stabilization problems.

A new high-gain observer able to overtake some of the drawbacks of classical structures has been recently proposed in Astolfi and Marconi (2015). In that paper, it is shown how to design a high-gain observer of dimension $2n-2$ for observable nonlinear systems with dimension n , which implements only gains proportional to ℓ and ℓ^2 while preserving the same behaviours of a standard high-gain observer. The new construction relies on an interconnected cascade of $n-1$ high-gain observers of dimension two.

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This observer practically solves the aforementioned challenging problem of numerical implementation present in standard high-gain observers. Moreover, it has been shown that the new observer structure substantially improves the sensitivity to high-frequency measurement noise. The proof of this fact has been presented in [Astolfi and Marconi \(2015\)](#) only for linear systems, and shown by numerical simulation in the nonlinear case. The new low-power high-gain observer has been also shown to be effective for a much wider class of nonlinear systems, such as system possessing a non-strict feedback form, see [Wang, Astolfi, Marconi, and Su \(2017\)](#). It turns out that the new observer structure is effective in all those frameworks where standard high-gain observers are typically used, such as output feedback stabilization by nonlinear separation principle and output regulation, [Astolfi, Isidori, and Marconi \(2017\)](#). Although the new observer structure solves the problem of numerical implementation, the peaking phenomenon is still present. This has been partially solved in [Astolfi, Marconi, and Teel \(2016\)](#), by adding saturations at various levels in the observer structure. With the proposed technique, it is possible to remove the peaking from the first $n - 1$ state estimates. Along this route, two similar schemes, which follow the seminal idea presented in [Astolfi and Marconi \(2015\)](#), have been recently proposed, in [Teel \(2016\)](#) and [Khalil \(2017\)](#), to address the implementation issues and the peaking phenomenon. In [Teel \(2016\)](#), the author shows how to build a high-gain observer by interconnecting a cascade of reduced order high-gain observer of dimension 1. A simpler scheme, without feedback interconnection terms, that cannot ensure asymptotic estimate, is presented in [Khalil \(2017\)](#). It is worth stressing, however, that even if the dimension of the observers is n , neither scheme improves the sensitivity properties with respect to standard high-gain observers.

The objective of this work is twofold. On the one hand, we combine the recent ideas of [Astolfi and Marconi \(2015\)](#) and [Astolfi et al. \(2016\)](#) to propose an observer of dimension $2n - 1$ which is still “low power” (namely it uses only gains proportional to ℓ and ℓ^2) and yet eliminates the peaking phenomenon. This is achieved by appropriately adding saturation functions in the observer dynamics. In particular, the n estimates provided by the proposed observer are peaking-free while the additional $n - 1$ auxiliary variables may reach values proportional to ℓ (and not to ℓ^{n-1} as in standard high-gain observers) during the transient. The resulting gain choices and transient behaviours address the numerical challenge. On the other hand, we fully characterize the sensitivity to high-frequency measurement noise for nonlinear systems by showing the improvement with respect to standard high-gain observers. This is done by extending the analysis tool recently introduced in [Astolfi, Marconi, Praly, and Teel \(2016\)](#) in which the sensitivity to measurement noise has been characterized for standard high-gain observers. In this work, for the sake of simplicity, we focus on the same class of nonlinear systems in canonical observability form considered in [Astolfi and Marconi \(2015\)](#), but similar results hold for the wider class of systems in feedback form ([Wang et al., 2017](#)).

The paper is organized as follows. We present the framework and we recall the high-gain observer technique in Section 2. Then, we provide the main results in Section 3. A simulation example is given in Section 4. The proofs of the main results are detailed in Section 5. Conclusions are discussed in Section 6. Some technical lemmas are given in [Appendix](#).

Notation. \mathbb{R} denotes the field of real numbers and, for $x \in \mathbb{R}^n$, $|x|$ denotes the Euclidean norm of x . With $s : \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}^m$ a bounded signal, we define $\|s\|_a^b := \sup_{t \in [a,b]} |s|$ and $\|s\|_\infty := \|s\|_0^\infty$. For $i > 0$ we denote by $A_i \in \mathbb{R}^{i \times i}$, $B_i \in \mathbb{R}^{i \times 1}$, $C_i \in \mathbb{R}^{1 \times i}$ a triplet in prime form, namely

$$A_i = \begin{pmatrix} 0_{i-1,1} & I_{i-1} \\ 0 & 0_{1,i-1} \end{pmatrix}, \quad B_i = \begin{pmatrix} 0_{i-1,1} \\ 1 \end{pmatrix}, \quad C_i^T = \begin{pmatrix} 1 \\ 0_{1,i-1} \end{pmatrix},$$

where $0_{i,j}$ denotes a matrix of dimension $i \times j$ containing zeros everywhere, and I_i denotes the identity matrix of dimension i . For $r > 0$, a saturation function $\text{sat}_r : \mathbb{R} \rightarrow \mathbb{R}$ is any strictly increasing C^1 function satisfying

$$\text{sat}_r(s) := s \quad \forall |s| \leq r, \quad |\text{sat}_r(s)| \leq r + 1 \quad \forall s \in \mathbb{R}.$$

With $C_{[0,1]}$ we denote the set of continuous functions from \mathbb{R} to $[0, 1]$.

2. The framework and highlights on high-gain observers

In this paper we deal with nonlinear single-input single-output systems that can be written, maybe after a change of coordinates, in the so-called *phase-variable form* (see [Gauthier and Kupka \(2001\)](#))

$$\begin{aligned} \dot{x}_i &= x_{i+1}, & i &= 1, \dots, n-1, \\ \dot{x}_n &= \varphi(x, d(t)) \\ y &= x_1 + v(t) \end{aligned} \quad (1)$$

where $x = (x_1, \dots, x_n)^T \in \mathbb{R}^n$ is the state, y is the measured output with v an additive unknown measurement noise, and $t \mapsto d(t) \in \mathbb{R}^{n_d}$, $n_d > 0$, is any (unknown) bounded signal that may represent parametric uncertainties in the function $\varphi(\cdot, \cdot)$ or unknown disturbances. The following assumption is made throughout the paper.

Assumption 1. The compact sets $D \subset \mathbb{R}^{n_d}$ and $X \subset \mathbb{R}^n$ and the positive $\tilde{\varphi}_x > 0$ are such that

- $d(t) \in D$ and $x(t) \in X$ for all $t \geq 0$;
- $|\varphi(x_1, d) - \varphi(x_2, d)| \leq \tilde{\varphi}_x |x_1 - x_2|$ for all $x_1, x_2 \in X$ and for all $d \in D$.

We observe that all the forthcoming analysis could be extended, with the appropriate modifications, to the case in which the function $\varphi(\cdot, \cdot)$ takes the form $\varphi(x, d, t)$ where the dependence on t takes into account the effect of possible known inputs. For sake of simplicity, however, we do not consider this case.

In the previous framework, we are interested in the semi-global high-gain observation problem, namely in the design of an asymptotic observer with a rate of convergence that can be made arbitrarily fast by tuning a single parameter (see [Khalil and Praly \(2014\)](#) and references therein).

The standard high-gain observer for the class of systems (1) is given by

$$\begin{aligned} \dot{\hat{x}}_i &= \hat{x}_{i+1} + k_i \ell^i e_1, & i &= 1, \dots, n-1, \\ \dot{\hat{x}}_n &= \varphi_s(\hat{x}) + k_n \ell^n e_1, \end{aligned} \quad (2a)$$

in which $\hat{x} = (\hat{x}_1, \dots, \hat{x}_n)^T$ is the state, ℓ is the high-gain parameter, e_1 is the output injection term defined as

$$e_1 := y - \hat{x}_1, \quad (2b)$$

k_1, \dots, k_n are design coefficients and $\varphi_s(\cdot)$ is any locally Lipschitz bounded function that agrees with $\varphi(\cdot, 0)$ on a compact set $X' \supset X$, namely $\varphi_s(x) = \varphi(x, 0)$ for all $x \in X'$ and for all $t \geq 0$. The tuning of the observer involves choosing the design parameters k_i 's so that, having defined the vector $K := \text{col}(k_1, \dots, k_n)$, the matrix $A_n - KC_n$ is Hurwitz, and taking the high-gain parameter ℓ large enough in relation to the Lipschitz constant of $\varphi(\cdot, \cdot)$ on $X \times D$. In particular, under [Assumption 1](#), it is possible to prove that, by letting $\ell^* := 2 \tilde{\varphi}_x |P|$, in which P is the symmetric positive definite matrix solution of the Lyapunov equation

$$P(A_n - KC_n) + (A_n - KC_n)^T P = -I,$$

then for all $\ell \geq \ell^*$ the estimation errors provided by the observer (2) satisfy the following bounds for all $t \geq 0$

$$\begin{aligned} |\hat{x}_i(t) - x_i(t)| &\leq c_1 \ell^{i-1} \exp(-c_2 \ell t) |\hat{x}(0) - x(0)| \\ &\quad + \frac{c_3}{\ell^{n+1-i}} \|d\|_\infty + c_4 \ell^{i-1} \|v\|_\infty \end{aligned} \quad (3)$$

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