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Decentralized control scheme for large-scale systems defined over a graph in presence of communication delays and random missing measurements[☆]

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ABSTRACT

This paper studies the decentralized output-feedback control of large-scale systems defined over a directed connected graph with communication delay and random missing measurements. The nodes in the graph represent the subsystems, and the edges represent the communication connection. The information travels across an edge in the graph and suffers from one step communication delay. For saving the storage space, the information delayed more than D step times is discarded. In addition, to model the system in a more practical case, we assume that the observation for the subsystem output suffers random missing. Under this new information pattern, the optimal output-feedback control problem is non-convex, what is worse, the separation principle fails. This implies that the optimal control problem with the information pattern introduced above is difficult to solve. In this paper, a new decentralized control scheme is proposed. In particular, a new estimator structure and a new controller structure are constructed, and the gains of the estimator and the controller are designed simultaneously. An optimality condition with respect to the gains is established. Based on the optimality condition, an iterative algorithm is exploited to design the gains numerically. It is shown that the exploited algorithm converges to Nash optimum. Finally, the proposed theoretical results are illustrated by a physical system which is a heavy duty vehicles platoon.

1. Introduction

Recently, much research attention has been paid to large-scale systems in which subsystems exchange information through a communication network, usually wireless. Such systems can be found in engineering fields, such as smart grids (Aldeen, Saha, Alpcan, & Evans, 2015), smart vehicle formations (Fax & Murray, 2004), and sensor network (Sivakumar, Sadagopan, & Baskaran, 2016). One feature of such systems is that the system performance is severely affected by the imperfections of the communication network (Heemels, Teel, Van de Wouw, & Nesic, 2010), such as packet losses, network delay, and communication constraint. To understand and counteract the effects of the communication network imperfections is becoming increasingly important. Especially, how to counteract the effects induced by the network delay for large-scale systems is a hot research topic.

Decentralized state-feedback linear quadratic Gaussian (LQG) control for large-scale systems defined over a directed connected graph with communication delay has been studied in Feyzmahdavian, Alam, and Gattami (2012), Lamperski and Doyle (2012), Lamperski and Lessard (2015), Matni and Doyle (2013). The authors of Lamperski and Doyle (2012) and Lamperski and Lessard (2015) designed an explicit optimal state-feedback LQG controller based on the independence decomposition of the process noise history. The varying communication delay case was investigated in Matni and Doyle (2013). The result of Matni and Doyle (2013) is only suitable for two-player systems. The design methods of Lamperski and Doyle (2012), Lamperski and Lessard (2015) and Matni and Doyle (2013) cannot be extended to the output-feedback case. The reason is that the independence decomposition for the measurements is not valid. In addition, the results of Lamperski and Doyle (2012), Lamperski and Lessard (2015) and Matni and Doyle (2013) were established under the assumption that the process noises of different subsystems are independent of each other. Removing this assumption, the explicit optimal state-feedback controller was

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found by the vectorization technique in Feyzmahdavian, Alam et al. (2012). The result of Feyzmahdavian, Alam et al. (2012) was extended to the output-feedback case in Feyzmahdavian, Gattami, and Johansson (2012). However, the result of Feyzmahdavian, Gattami et al. (2012) is only for the three-player systems with chain structure, and is unlikely to be extended to the large-scale systems composed of N subsystems. For large-scale systems composed of N subsystems, decentralized output feedback control with delay sharing pattern was investigated in Kurtaran and Sivan (1974) and Nayyar, Mahajan, and Teneketzis (2011). The authors of Kurtaran and Sivan (1974) designed an optimal output feedback LQG controller under one step delay sharing pattern by dynamic programming. Two structural results for multiple step delays sharing pattern were established in Nayyar et al. (2011). In addition, the decentralized output feedback controller with asymmetric one step delay sharing pattern was designed in Nayyar, Kalathil, and Jain. (2018). However, for the delay model defined over a directed connected graph, the decentralized output feedback control of large-scale systems composed of N subsystems is not fully studied.

On the other hand, in Feyzmahdavian, Gattami et al. (2012), Kurtaran and Sivan (1974) and Nayyar et al. (2018, 2011), it is assumed that the observation for the subsystem output is always valid. However, the observation may be affected by uncertain factors in engineering practice, and thus may suffer from random missing. For systems with random missing measurements (uncertain observation), the linear filtering problems have been studied, see (Ma & Sun, 2011; Moayedi, Foo, & Soh, 2010); however, the decentralized controller design considering random missing measurements is still an open problem. To design the optimal decentralized controller under random missing measurement is a challenge task, because the separation principle (Yoshikawa & Kobayashi, 1978) may fail.

In this paper, we focus on the decentralized output feedback LQG control for large-scale systems with communication delays and random missing measurements. The large-scale system is composed of N subsystems, and is defined over a directed connected graph. The nodes in the graph represent the subsystems. The measurement output in each subsystem contains valid measurement or noise only (random missing measurements). The edges in the graph represent the communication network. The information travels across an edge with one step delay. Such a delay model was introduced in Lamperski and Doyle (2012) and Lamperski and Lessard (2015), and was applied to vehicle formations control in Feyzmahdavian, Alam et al. (2012). In this paper, it is assumed that each subsystem maintains a buffer of length $D+1$ such that the information delayed more than D step times is discarded. Under this setup, the corresponding optimal LQG control problem is non-convex. To solve this optimal control problem, we propose a new decentralized control scheme. Firstly, a new estimator structure and a new controller structure are constructed. It is shown that the separation principle (Yoshikawa & Kobayashi, 1978) fails. Secondly, an optimality condition with respect to the gains of the estimator and the controller is established. Thirdly, we give an iterative algorithm to find the gains of the estimator and the controller simultaneously, and we show that the algorithm converges to Nash optimum. Lastly, we use a heavy duty vehicles platoon to illustrate the theoretical results proposed in this paper.

Notation. For a directed graph $\mathcal{G}=(\mathcal{V},\mathcal{E})$, $\mathcal{V}=\{1,\dots,N\}$ is the node set and $\mathcal{E}\subset\mathcal{V}\times\mathcal{V}$ is the edge set; define $\mathcal{N}_i=\{j:(j,i)\in\mathcal{E}\}$, where (j,i) is an arrow from j to i. The sequence $\{x_0,\dots,x_t\}$ is denoted by $x_{0:t}$. Let $\mathrm{tr}(X)$ denote the trace of the square matrix X. $\mathbb{E}(x)$ is the expectation of the random variable x, and $\mathbb{E}(x|y)$ is the conditional expectation of x given y. Let A^{T} denote the transpose of the matrix A. The notations $X\succ 0$ and $X\succeq 0$ mean that X is a positive definite matrix and a positive semi-definite matrix, respectively. The $m\times n$ zero matrix is denoted by $0_{m\times n}$, and the $n\times n$ zero matrix is denoted by 0_n. For a matrix A, $(A)^n$ denotes the nth power of A, and $(A)_{ij}$ denotes the ith row, jth column element of A. $\Pr(\cdot)$ is the probability measure. For two sets $\mathbb{X}_1$, $\mathbb{X}_2$, define $\mathbb{X}_1\setminus\mathbb{X}_2\triangleq\{x:x\in\mathbb{X}_1 \text{ and } x\notin\mathbb{X}_2\}$.

2. Problem statement

Consider a large-scale system defined over a connected directed graph $\mathcal{G}=(\mathcal{V},\mathcal{E})$ with $\mathcal{V}=\{1,\dots,N\}$, where the nodes represent the subsystems. The ith subsystem is of the form

$$x_{t+1}^i = A^{ii}x_t^i+\sum_{j\in\mathcal{N}_i}A^{ij}x_t^j+B^iu_t^i+\omega_t^i, \quad (1)$$

$$y_t^i=\delta_t^iC^ix_t^i+\upsilon_t^i. \quad (2)$$

For the ith subsystem, $x^i\in\mathbb{R}^{n_i}$ is the state; $u^i\in\mathbb{R}^{l_i}$ is the control input; $y^i\in\mathbb{R}^{m_i}$ is the measurement output; $\omega^i\in\mathbb{R}^{n_i}$ is the process noise; $\upsilon^i\in\mathbb{R}^{m_i}$ is the measurement noise. The matrices A^{ij}, B^i and C^i are of proper dimensions for all $i,j\in\mathcal{V}$; δ_t^i is a random binary variable, and satisfies

$$\Pr(\delta_t^i=1)=\lambda,\quad \Pr(\delta_t^i=0)=1-\lambda,$$

where $\delta_t^i=0$ means that the measurement of the ith subsystem is missing (the output contains noise only); $\delta_t^i=1$ implies that the measurement of the ith subsystem is valid. It is assumed that $\delta_{t_1}^i$ is independent of $\delta_{t_2}^j$ for any $t_1\neq t_2$ or $i\neq j$.

Define the following matrices

$$A=\begin{bmatrix}A^{11} & \cdots & A^{1N}\\ \vdots & \ddots & \vdots\\ A^{N1} & \cdots & A^{NN}\end{bmatrix},\ A^{ij}=0 \text{ for } (i,j)\notin\mathcal{E},$$

$$B=\begin{bmatrix}B^1 & & \\ & \ddots & \\ & & B^N\end{bmatrix},\ C=\mathrm{diag}\{C^1,\dots,C^N\},$$

$$\delta_t=\mathrm{diag}\{\delta_t^1I_{m_1},\dots,\delta_t^NI_{m_N}\}.$$

Stacking x^i, u^i, ω^i, υ^i and y^i into augmented vectors

$$x=\begin{bmatrix}x^1\\ \vdots\\ x^N\end{bmatrix},\quad u=\begin{bmatrix}u^1\\ \vdots\\ u^N\end{bmatrix},\quad y=\begin{bmatrix}y^1\\ \vdots\\ y^N\end{bmatrix},$$

$$\omega=\begin{bmatrix}\omega^1\\ \vdots\\ \omega^N\end{bmatrix},\quad \upsilon=\begin{bmatrix}\upsilon^1\\ \vdots\\ \upsilon^N\end{bmatrix}.$$

The large-scale system (1)–(2) can be written as

$$x_{t+1}=Ax_t+Bu_t+\omega_t, \quad (3)$$

$$y_t=\delta_tCx_t+\upsilon_t, \quad (4)$$

where the initial state x_0 is a Gaussian variable with $x_0\sim\mathcal{N}(\bar{x},\Theta_0)$, $\Theta_0\succ 0$. The noises ω_t and υ_t are the independent Gaussian processes with $\omega_t\sim\mathcal{N}(0,W_t)$, $W_t\succ 0$. and $\upsilon_t\sim\mathcal{N}(0,V_t)$, $V_t\succ 0$, respectively. Assume that x_0, ω_{t_1} and υ_{t_2} are pairwise independent for all t_1,t_2. In addition, the system parameters A, B, C, Θ_0, W_t and V_t are known to all subsystems.

The information pattern in this paper is described as follows. The information travels across an edge in the graph and requires one step time. Define τ_{ij} as the length of the shortest path from the ith subsystem to the jth subsystem, where $\tau_{ii}=0$. Hence,