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Prediction error identification of linear dynamic networks with rank-reduced noise☆

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ABSTRACT

Dynamic networks are interconnected dynamic systems with measured node signals and dynamic modules reflecting the links between the nodes. We address the problem of identifying a dynamic network with known topology, on the basis of measured signals, for the situation of additive process noise on the node signals that is spatially correlated and that is allowed to have a spectral density that is singular. A prediction error approach is followed in which all node signals in the network are jointly predicted. The resulting joint-direct identification method, generalizes the classical direct method for closed-loop identification to handle situations of mutually correlated noise on inputs and outputs. When applied to general dynamic networks with rank-reduced noise, it appears that the natural identification criterion becomes a weighted LS criterion that is subject to a constraint. This constrained criterion is shown to lead to maximum likelihood estimates of the dynamic network and therefore to minimum variance properties, reaching the Cramér–Rao lower bound in the case of Gaussian noise. In order to reduce technical complexity, the analysis is restricted to dynamic networks with strictly proper modules.

1. Introduction

It is becoming more common to model complex dynamic systems as networks of interconnected dynamic modules, or *dynamic networks*. Data-driven modeling, or *identification*, of modules in these dynamic networks is then a natural problem to address. Applications range over many fields, for example identification of dynamics that connect different (MPC) control loops in industrial process control (Gudi & Rawlings, 2006; Van den Hof, Dankers, & Weerts, 2018), identification of biochemical networks (Yuan, Stan, Warnick, & Gonçalves, 2011), modeling of the dynamic behavior of a ship as a dynamic network (Linder, 2017), and modeling of stock prices in financial markets as a dynamic network (Materassi & Innocenti, 2010).

Various approaches have been developed for identification of dynamic networks, roughly divided into three categories. The first approach considers the identification of a single module in the dynamic network in the situation that the interconnection structure, or topology, of the network is known. The second approach focusses on identification of the full network dynamics for a given topology, and the last category deals with the identification of the topology (and dynamics) of the network. For identification of single modules, authors have used e.g. Wiener filters (Materassi & Salapaka, 2012), while the estimation of parametric transfer functions in a prediction error setting has been addressed in Dankers, Van den Hof, Bombois, and Heuberger (2015), Dankers, Van den Hof, Heuberger, and Bombois (2016), Gevers and Bazanella (2015), Linder and Enqvist (2017) and Van den Hof, Dankers, Heuberger, and Bombois (2013). Identification of the full network dynamics has been considered by modeling the network as a state–space system (Haber & Verhaegen, 2014), or as a network of transfer function modules (Weerts, Van den Hof, & Dankers, 2016b). Identifiability properties related to this problem have been addressed in Adebayo et al. (2012), Gevers, Bazanella, and Parraga (2017), Gonçalves and Warnick (2008), Weerts, Dankers, and Van den Hof (2015) and Weerts, Van den Hof, and Dankers (2018). Some different methods for topology detection can be found in literature, for example following a Bayesian approach (Chiuso & Pillonetto, 2012), a compressed sensing approach

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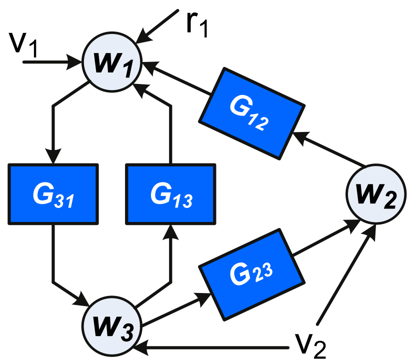

Fig. 1. Example of a network with rank-reduced noise. Node signals are w_i, being the outputs of the (circular) summation points, interconnected by modules G_{ij} and perturbed by non-measured disturbance signals v_i. Signals r_i are excitation signals available to the user.

(Hayden, Chang, Gonçalves, & Tomlin, 2016), or through one-step ahead prediction using Wiener filters (Materassi & Salapaka, 2012).

In this paper we consider networks that consist of measured node signals, which are interconnected by linear dynamic modules, as depicted in Fig. 1, and in line with the setup as defined in Van den Hof et al. (2013). We will address the problem of identifying, on the basis of measured node signals, the dynamics of all modules in a network, of which the topology is known, and where conditions on the disturbance signals v_i in the network are more general than typically considered. While in the current literature it is usually assumed that every node signal in the network has a non-zero process noise v_i that is uncorrelated to all other noises, i.e. for the vector noise process v it holds that $\Phi_v(\omega)$ is diagonal, we will address two steps of generalization:

- We will allow noise signals on the different node signals to be spatially correlated, i.e. $\Phi_v(\omega)$ is not necessarily diagonal, and
- We will allow $\Phi_v(\omega)$ to be singular, implying that node signals can be noise-free, or that disturbances are exactly related with each other through a linear filter.

Concerning the first step, this situation includes the handling of confounding variables, i.e. unmeasured variables that affect both inputs and outputs of an estimation problem. This notion is widely used in statistical estimation problems in networks and is also used in network identification problems, Dankers et al. (2016). The relation between confounding variables and correlated disturbances has been explained in Van den Hof, Dankers, and Weerts (2017).

Concerning the second step, note that modules in a network can also be implemented controllers, and controller outputs can be noise-free, as e.g. typically considered in a classical closed-loop identification problem (Ljung, 1999). In this case there is no process noise on a particular node signal. Alternatively, strong correlations between disturbance signals can occur e.g. if the network is a spatially distributed system affected by global disturbances, like a wind gust affecting wind turbines in a wind park. A deterministic relation between disturbance signals (like e.g. a delay) will cause the full disturbance spectrum to lose its full rank. A situation of loss of full rank is depicted in Fig. 1 where the process noises on nodes 2 and 3 are the same (perfect correlation). When identifying the full network dynamics, aiming not only at consistency of the module estimates, but also at minimum variance results, correlated disturbances will prevent the identification problem to be decomposable into separate multi-input single-output problems. The fact that the noise process is allowed to be rank-reduced causes some fundamental issues that need to be addressed in the prediction error identification setting.

Identification in the situation of rank-reduced noise is a topic that has not been widely addressed in the prediction error identification literature. Dynamic factor models have been developed in Deistler, Scherrer, and Anderson (2015) and Felsenstein (2014) to deal with rank-reduced noise. Maximum likelihood estimates with rank-reduced noise have been obtained for vector autoregressive systems (Kölbl, 2015) and linear regression (Srivastava & von Rosen, 2002). In a prediction error setting, the property of *network identifiability* has been defined in Weerts et al. (2015) and Weerts et al. (2018), covering also the situation of rank-reduced noise, while predictor models have been analyzed for the situation of noise-free nodes in Weerts, Van den Hof, and Dankers (2016a). In Weerts, Van den Hof, and Dankers (2017) a first analysis of consistent estimation of network models has been presented for the reduced-rank noise case, leading to the use of weighted and constrained least-squares identification criteria. This was a further extension of the preliminary work of Van den Hof, Weerts, and Dankers (2017b) where an open-loop one-input two-output situation with rank-reduced output noise was considered.

In this paper we are going beyond the consistency question, by including an analysis of the asymptotic variance of the prediction error method, and by developing the maximum likelihood estimator and the Cramér–Rao lower bound on the variance, for the situation of correlated and rank-reduced noise, while addressing networks with strictly proper modules. This paper builds on and further extends the preliminary results of Weerts et al. (2017).

First a definition of the dynamic network setup and the rank-reduced noise process is given in Section 2. Then, in Section 3, the prediction error identification setup is presented and a least squares identification criterion is shown to provide consistent estimates. In Section 4 the dependencies in the noise process are explicitly used to construct a constrained least squares identification criterion that is shown to lead to a maximum likelihood estimate under some conditions. An analysis of the asymptotic variance of the estimates is made in Section 5, where the variance expressions are related to the Cramér–Rao lower bound. Finally in Section 6 the theoretical results are illustrated in a numerical simulation example.

2. Dynamic network definition

Following the basic setup of Van den Hof et al. (2013), a dynamic network is built up out of L scalar *internal variables* or *nodes* w_j, $j = 1, \ldots, L$, and K *external variables* r_k, $k = 1, \ldots, K$. Each internal variable is described as:

$$w_j(t) = \sum_{\substack{l=1 \\ l\neq j}}^{L} G^0_{jl}(q)w_l(t) + \sum_{k=1}^{K} R^0_{jk}(q)r_k(t) + v_j(t) \quad (1)$$

where q^{-1} is the delay operator, i.e. $q^{-1}w_j(t) = w_j(t-1)$;

- G^0_{jl} are strictly proper rational transfer functions, and the single transfers G^0_{jl} are referred to as *modules* in the network.
- r_k are *external variables* that can directly be manipulated by the user, and R^0_{jk} are proper rational transfer functions;
- v_j is *process noise*, where the vector process $v = [v_1 \cdots v_L]^T$ is modeled as a stationary stochastic process with rational spectral density, such that there exists a p-dimensional white noise process $e := [e_1 \cdots e_p]^T$, $p \leq L$, with covariance matrix $\Lambda^0 > 0$ such that

  $$v(t) = H^0(q)e(t),$$

with $H^0(q)$ a proper rational transfer function.