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Brief paper

# Modeling and iterative pulse-shape control of optical chirped pulse amplifiers[☆]

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ABSTRACT

In this paper, we present an iterative learning algorithm for pulse-shape control applications of optical chirped pulse amplifiers for ultra-short, high-energy light pulses. For this, we first introduce a general nonlinear and infinite-dimensional mathematical model of chirped pulse amplifiers. By reducing the complexity of this detailed model and reformulating the control task, we are subsequently able to apply inversion-based iterative learning control to track desired output pulses. Using the reduced model to estimate both internal states and unknown parameters yields a fast and simple way of consistently estimating the input–output behavior without relying on a calibrated system model. The effectiveness of the resulting adaptive algorithm is finally illustrated with simulation scenarios on an experimentally validated mathematical model.



## 1. Introduction

Over the last several decades, the systematic generation, detection, and manipulation of light opened a novel and rapidly growing field of research, now commonly known as photonics. However, the contributions of control engineering to this field are comparatively scarce with some exceptions like the control of mode-locked lasers (Brunton, Fu, & Kutz, 2013), pulse shaping (Ren, Frihauf, Krstic, & Rafac, 2011; Ren, Frihauf, Rafac, & Krstic, 2012; Webb, Ellison, Desbruslais, Fedoruk, & Turitsyn, 2009) and pulse propagation (Omenetto, Reitze, Luce, Moores, & Taylor, 2002). One particular task in photonic applications is the amplification of light pulses, especially for ultra-short high-energy pulses as used in strong field physics (Krausz & Ivanov, 2009), for coherent control (Goswami, 2003) or for pumping of optical parametric amplifiers (Malevich et al., 2013). The amplification of high-energy pulses is usually done by multipass amplification (Lowdermilk & Murray, 1980) using regenerative amplifiers (RAs), where a (usually) continuously pumped gain medium is placed inside an optical resonator. The pulse is then cycled several times until the stored energy of the gain medium is extracted and the amplified pulse is released. Since the amplification of intense laser pulses in active gain media is limited in its maximum energy due to self-focusing effects, a common technique to amplify high-energy pulses is to use so-called optical chirped pulse amplifiers (CPAs) (Strickland & Mourou, 1985). The idea of CPAs is to stretch the incident pulse to reduce its power density and amplify the stretched pulse. This stretching can be achieved by introducing large amounts of artificial positive dispersion and thus convert the incident pulse into a chirped pulse. Afterwards, one wishes to recompress the chirped pulse by applying negative dispersion (Treacy, 1969). While this is possible for RAs with spectrally uniform gain, the quality of the amplified pulse is degrading rapidly for non-uniform gain which is typically the case for broad-band amplifiers needed for ultra-short pulse amplification. To approximately compensate for non-uniform gain and resulting effects like gain-narrowing (Hotz, 1965), spectral filters in front of the RA (Malevich et al., 2013) or within the resonator (Kroetz et al., 2016) have been successfully applied to achieve spectrally broad and thus temporally short pulses after recompression. However, these filters have to be adapted individually for each point of operation. The availability of programmable spectral filters for pulse shaping (Tournois, 1997; Weiner, 2011) makes the compensation by automatic control strategies a desirable option.

In general, the applications of control theory to problems in photonics mostly utilize model-free concepts like extremum-seeking (ES) (Brunton et al., 2013; Ren et al., 2012) or some kind of genetic algorithm (Omenetto et al., 2002) or model-free versions of iterative learning control (ILC) (Ren et al., 2011). This is typically

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reasoned by two arguments, namely the degree of uncertainty inherent to some parameter values and the highly complex effects of the nonlinear and infinite-dimensional dynamics involved. In Ren et al. (2011), a PD-type ILC is used to find the input pulse necessary to obtain a desired output pulse for an (unchirped) single-pass amplifier with spectrally uniform gain. Unlike such amplifiers, the CPA systems considered in this paper exhibit a variety of effects that make them quite challenging from a control point of view, e.g., extremely high gain levels that are saturating in a spectrally inhomogeneous way and nonlinear dispersive effects of the pulse propagation. Additionally, CPAs are typically operated in a regime where subsequent pulses are coupled as the stored energy of the gain medium is not allowed to fully recover in between two pulses. This can even induce an unstable system behavior (Grishin, Gulbinas, & Michailovas, 2007). From a control perspective, CPAs constitute a challenging class of systems. Accordingly, the main contribution of this paper lies in the introduction of a novel and challenging application to the control audience, the derivation of a comprehensive mathematical model and the development and evaluation of tailored ILC concepts on an experimentally validated model.

In this paper, we present a pulse-shape control for optical CPAs to track desired output pulses by means of inversion-based ILC. For this, we start by deriving a detailed nonlinear mathematical model of the process describing the evolution of the light field in Section 2. Section 3 then deduces a simplified linear model upon which the inversion-based ILC strategy is developed. The effectiveness of the inversion-based ILC strategy is then verified by simulation scenarios in Section 4 and some final conclusions are drawn in Section 5.

### 1.1. Mathematical framework and nomenclature

Before beginning to derive a complete mathematical model of the CPA, some preliminary statements will be made. In ultra-fast optics it is common to represent field quantities as real parts of complex quantities $\mathcal{A}$ that are described by complex envelope representations. For a plane wave with fixed polarization propagating along the $z$ axis, this can be written as $\mathcal{A}(z,t) = A(z,t)\mathrm{e}^{-i(k_0 z-\omega_0 t)}$, with the complex pulse envelope $A(z,t)$ of a carrier wave, the time $t$, the spatial coordinate $z$, the imaginary unit $i$, the angular frequency $\omega_0$ and the spatial wave number $k_0$. Since the pulses of interest are typically signals where the spatial and temporal variations of the complex pulse envelope are slow compared to the carrier oscillations, the approximations $\left|\frac{\partial^2 A(z,t)}{\partial z^2}\right| \ll \left|k_0 \frac{\partial A(z,t)}{\partial z}\right|$ and $\left|\frac{\partial^2 A(z,t)}{\partial t^2}\right| \ll \left|\omega_0 \frac{\partial A(z,t)}{\partial t}\right| \ll \left|\omega_0^2 A(z,t)\right|$ usually called slowly varying envelope approximation (SVEA), see, e.g., Reider (2016) or Siegman (1986, Sec. 24.4), are applied. To analyze a pulse spectrally, the Fourier transform $\hat{\mathcal{A}}(z,\omega) = \mathfrak{F}\{\mathcal{A}(z,t)\}$ is used. All considered pulses at some location $z_0$ are bounded and of finite energy and thus $\mathcal{A}(z_0,\cdot) \in L^2 \cap L^\infty$ with the common norms

$$\|\mathcal{A}(z_0,\cdot)\|_2^2 = \|A(z_0,\cdot)\|_2^2 = \int_{-\infty}^{\infty} |A(z_0,t)|^2 \mathrm{d}t \tag{1a}$$

$$\|\mathcal{A}(z_0,\cdot)\|_\infty = \|A(z_0,\cdot)\|_\infty = \sup_{t\in\mathbb{R}} |A(z_0,t)| \,. \tag{1b}$$

Following the convention to describe the light pulse using the electric field $\mathcal{E}$, its pulse energy at $z_0$ is given by $W = {}^{A_B}/_{2Z_0}\|E(z_0,\cdot)\|_2^2 = {}^{A_B}/_{4\pi Z_0}\|\hat{E}(z_0,\cdot)\|_2^2$, where $Z_0$ denotes the impedance of free space and $A_B$ is the cross section of the laser beam. In the sequel, spatial mean values over the length of the gain medium $L$ are denoted by an overline, i.e. $\overline{N} = {}^1/_L \int_0^L N(z)\mathrm{d}z$.

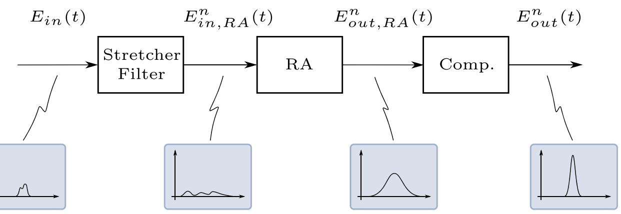


**Fig. 1.** Components of a CPA system and associated pulses. The schematic graphs in blue illustrate the ideal temporal pulse shapes for a desired Gaussian output pulse.

## 2. Mathematical model

Apart from the source of input pulses, a CPA system consists of three main components: a pulse stretcher with a spectral filter afterwards, a regenerative amplifier (RA) and a pulse compressor at the end. Such CPA systems are usually operated in a repetitive fashion, where identical seed pulses $E_{\mathrm{in}}(t)$ at a repetition frequency $f_{\mathrm{rep}}$ are fed into a (typically grating-based) pulse stretcher. Depending on the settings of the spectral filter and the state of the RA, this gives a heavily chirped pulse $E^n_{\mathrm{in,RA}}(t)$ which is amplified to $E^n_{\mathrm{out,RA}}(t)$ and compressed back to yield the output pulse $E^n_{\mathrm{out}}(t)$. The overall goal is to adjust the amplitude and phase characteristic of the spectral filter such that $E^n_{\mathrm{out,RA}}(t)$ is compressed back into an unchirped pulse of desired shape $E^{\mathrm{d}}_{\mathrm{out}}(t)$ as indicated in Fig. 1. While pulse stretchers and compressors can be easily described in terms of their spectral properties, the behavior of RAs is quite complex and requires a more detailed model. Thus, we start by addressing the description of the stretcher, the compressor and the spectral filter in Section 2.1 and then continue with the RA in Section 2.2.

### 2.1. Stretcher, filter and compressor

The basic idea of grating-based pulse stretchers (and compressors) is to vary the spatial paths depending on the frequency and thus introduce frequency-depending delays. When propagating through the device, each frequency component $\omega$ of the complex envelope's Fourier transform $\hat{E}(z,\omega)$ thus receives an additional phase $\varphi_{\mathrm{S}}(\omega)$ that can be approximated around the carrier frequency $\omega_0$ with high accuracy by

$$\varphi_{\mathrm{S}}(\nu) = \varphi_{\mathrm{S},0} + \varphi_{\mathrm{S},1}\nu + \varphi_{\mathrm{S},2}\nu^2 + \varphi_{\mathrm{S},3}\nu^3 \tag{2}$$

where $\nu = \omega - \omega_0$. The constants $\varphi_{\mathrm{S},i}$, $i \in \{0,1,2,3\}$ depend on the geometry, the pulse's center frequency $\omega_0$ and the grating constant as shown in Treacy (1969). The transfer function of the pulse stretcher can therefore be written as $G_{\mathrm{S}}(\omega) = \eta_{\mathrm{S}}\mathrm{e}^{i\varphi_{\mathrm{S}}(\omega-\omega_0)}$ with the spectrally uniform efficiency coefficient $\eta_{\mathrm{S}}$. Thus, the input pulse to the RA is given by

$$E^n_{\mathrm{in,RA}}(t) = \mathfrak{F}^{-1}\left\{G^n_{\mathrm{F}}(\omega)\, G_{\mathrm{S}}(\omega)\, \hat{E}_{\mathrm{in}}(\omega)\right\}, \tag{3}$$

where $G^n_{\mathrm{F}}(\omega)$ denotes the adjustable transfer function of the spectral filter (set for the $n$th pulse). Since the filter is not able to amplify any frequency component, its transfer function has to fulfill the constraint

$$\left|G^n_{\mathrm{F}}(\omega)\right| \le 1 \tag{4}$$

for all $\omega \in \mathbb{R}$. Analogously, the compressor can be described by $E^n_{\mathrm{out}}(t) = \mathfrak{F}^{-1}\left\{G_{\mathrm{C}}(\omega)\, \hat{E}^n_{\mathrm{out,RA}}(\omega)\right\}$ with $G_{\mathrm{C}}(\omega) = \eta_{\mathrm{C}}\mathrm{e}^{i\varphi_{\mathrm{C}}(\omega-\omega_0)}$ and $\varphi_{\mathrm{C}}$ according to (2).

**Remark 1.** Analyzing the effect of (2) shows that the constant and linear terms introduce a phase shift and a time delay to the