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Brief paper

Nonlinear state estimation under bounded noises ☆

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ABSTRACT

Most of the existing nonlinear state estimation methods require to know the statistical information of noises. However, the statistical information may not be accurately obtained or satisfied in practical applications. Actually, the noises are always bounded in a practical system. In this paper, we study the nonlinear state estimation problem under bounded noises, where the addressed noises do not provide any statistical information, and the bounds of noises are also unknown. By using matrix analysis and second-order Taylor series expansion, a novel constructive method is proposed to find an upper bound of the square error of the nonlinear estimator. Then, a convex optimization problem on the design of an optimal estimator gain is established in terms of linear matrix inequalities, which can be solved by standard software packages. Moreover, stability conditions are derived such that the square error of the designed nonlinear estimator is asymptotically bounded. Finally, two illustrative examples are employed to show the advantages and effectiveness of the proposed methods.

1. Introduction

State estimation has attracted considerable research interest during the past decades, and has found applications in a variety of areas, including target tracking (Bar-Shalom, Li, & Kirubarajan, 2001), robot localization (Roumeliotis & Bekey, 2002), information fusion (Chen, Hu, Zhang, & Yu, 2016) and cyber–physical systems (Chen, Hu, Ho, & Yu, 2016). Generally, estimation performance is determined by the physical dynamical process and the noises. In most practical systems, the physical processes are nonlinear, while the noises are bounded. Since nonlinear systems with noises have complex dynamic processes, state estimation for nonlinear systems has been always a challenging problem. In this paper, we will focus on the design of a nonlinear estimator under bounded noises.

When considering Gaussian white noises with known covariances, there mainly exist three classical nonlinear estimation methods: extended Kalman filter (EKF) (Reif, Günther, Yaz, & Unbehauen, 1999), unscented Kalman filter (UKF) (Julier & Uhlmann, 1997, 2004) and cubature Kalman filter (CKF) (Arasaratnam & Haykin, 2009). Particularly, a multiple model extension to the EKF was proposed in Duan and Li (2015) to improve the estimation performance, while an uncorrelated conversion method was introduced in Lan and Li (2015) to design the nonlinear estimator in the sense of linear minimum mean square error. Furthermore, when considering non-Gaussian white noise with known probability density function, a Bayesian framework was developed in Gordon, Salmond, and Smith (1993) to design a nonlinear estimator, and the strategy of Gaussian sum reapproximation was introduced in Psiaki, Schoenberg, and Miller (2015) to develop a nonlinear estimation algorithm. Notice that the above nonlinear estimation methods are required to know the statistical properties of noises. However, these statistical properties may not be obtained or satisfied in practical applications. To overcome this weakness, by introducing the H_∞ filtering theory, the finite-horizon extended H_∞ estimator (EHE) and second-order EHE have been designed in Hu and Yang (2011) and Yang, Wang, Lauria, and Liu (2009) for a class of nonlinear systems with energy-bounded noises, where the

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sequence of the weighting matrices in the optimization objective were required to be given over finite-time horizon in advance, and could affect the estimation performance. Notice that the infinite-horizon nonlinear estimation cannot be obtained under the framework of EHEs, and thus the stability condition of EHEs may be difficult to be derived.

On the other hand, when the bounded noises are specified ellipsoidal sets, the set-membership nonlinear filtering algorithms have been developed in Calafiore (2005) and Yang and Li (2010), where the bounds of noises were required to be known in advance. Particularly, the linearization errors in Calafiore (2005) were modeled by the parameter uncertainties with known bounds, while the estimation performance in Yang and Li (2010) was dependent on the fuzzy rule, and the corresponding fuzzy errors were also modeled by the parameter uncertainties with known bounds. Meanwhile, the moving horizon estimation methods have been developed in Alessandri, Baglietto, and Battistelli (2008) and Müller (2017) for discrete-time nonlinear systems with bounded disturbances, where the weighting matrices in the optimization objective were required to be given in advance, and the stability conditions were dependent on the bounds of noises. Although several estimation algorithms have been developed in Alessandri et al. (2008), Calafiore (2005), Hu and Yang (2011), Müller (2017), Yang and Li (2010) and Yang et al. (2009) for nonlinear systems with bounded noises, there are still conservatism and drawbacks from the noise assumptions and the methods of dealing with nonlinear terms.

Motivated by the aforementioned analysis, a new state estimation method is developed in this paper for nonlinear systems with unknown but bounded noises, where the lower and upper bounds of noises are also unknown. Notice that the addressed noises are much more general, and can be easily satisfied in a practical system. A Kalman-like estimation structure is proposed to design nonlinear estimators. The contributions of this paper are summarized as follows: (i) By using second-order Taylor series expansion, a linear time-varying state-space model with Taylor approximation errors and system noises is developed to depict the nonlinear estimation errors, where the Taylor approximation error is viewed as a bounded disturbance at a particular time; (ii) An upper bound of nonlinear estimation error, which is dependent on the bounded noise and Taylor approximation error, is constructed at each time. By minimizing this upper bound at each time, a convex optimization problem on the design of nonlinear estimator is established in terms of linear matrix inequalities, which can be directly solved by standard software packages. Moreover, stability conditions are derived such that the square error of nonlinear estimator is asymptotically bounded. Finally, a numerical example and localization of a mobile robot are used to demonstrate the effectiveness of the proposed methods. Moreover, the advantages of the nonlinear estimation method in this paper are shown by comparing with the classical EKF (Reif et al., 1999), UKF (Julier & Uhlmann, 2004) and CKF (Arasaratnam & Haykin, 2009).

Notations: The superscript "T" represents the transpose, while "I" represents the identity matrix with appropriate dimension. $X > (<)0$ denotes a positive-definite (negative-definite) matrix, and diag$\{\cdot\}$ stands for a block diagonal matrix. $\lambda_{\max}(\cdot)$ means the maximum eigenvalue of the corresponding matrix, while $\|X\|_2$ is the 2-norm of matrix X. The symmetric terms in a symmetric matrix are denoted by "$*$", and col$\{a_1, \ldots, a_n\}$ means a column vector whose elements are $a_1, \ldots, a_n$. Moreover, if $\tau_1 > \tau_2$, it will be specified that $\prod_{\tau=\tau_1}^{\tau_2} F(\tau) = I$ and $\sum_{\tau=\tau_1}^{\tau_2} G(\tau) = 0$, where $F(\tau)$ and $G(\tau)$ represent different matrix functions with respect to the variable τ.

2. Problem formulation

Consider a nonlinear dynamical system described by the following state-space model:

$$\begin{cases} \mathrm{x}(t+1) = f(\mathrm{x}(t)) + \Gamma \mathrm{w}(t) \\ \mathrm{y}(t) = g(\mathrm{x}(t)) + \mathrm{v}(t) \end{cases} \tag{1}$$

where $\mathrm{x}(t) \in \mathrm{R}^n$ is the system state, $\mathrm{y}(t) \in \mathrm{R}^m$ is the measurement output. In (1), $f(\mathrm{x}(t)) \triangleq \mathrm{col}\{f_1(\mathrm{x}(t)), \ldots, f_n(\mathrm{x}(t))\} \in \mathrm{R}^n$ and $g(\mathrm{x}(t)) \triangleq \mathrm{col}\{g_1(\mathrm{x}(t)), \ldots, g_m(\mathrm{x}(t))\} \in \mathrm{R}^m$ are nonlinear vector functions that are assumed to be continuously differentiable, while Γ is a constant matrix with appropriate dimension, and $\mathrm{w}(t)$ and $\mathrm{v}(t)$ are unknown but bounded noises, i.e.,

$$\|\mathrm{w}(t)\|_2 \le \delta_{\mathrm{w}}, \ \|\mathrm{v}(t)\|_2 \le \delta_{\mathrm{v}} \tag{2}$$

where the bounds δ_{w} and δ_{v} are unknown. Then, the nonlinear estimator $\hat{\mathrm{x}}(t)$ is proposed to be given by Kalman-like structure:

$$\begin{cases} \hat{\mathrm{x}}^p(t) = f(\hat{\mathrm{x}}(t-1)) \\ \hat{\mathrm{x}}(t) = \hat{\mathrm{x}}^p(t) + \mathrm{K}(t)(\mathrm{y}(t) - g(\hat{\mathrm{x}}^p(t))) \end{cases} \tag{3}$$

where $\hat{\mathrm{x}}^p(t)$ denotes one-step prediction. Consequently, the problems to be solved in this paper are described as follows:

- The first aim is to design an optimal estimator gain $\mathrm{K}(t)$ such that an upper bound of the square error of the nonlinear estimator $\hat{\mathrm{x}}(t)$ is minimal at each time;
- The second aim is to find stability conditions such that the square error of the nonlinear estimator $\hat{\mathrm{x}}(t)$ is asymptotically bounded (i.e., $[\mathrm{x}(t) - \hat{\mathrm{x}}(t)]^{\mathrm{T}}[\mathrm{x}(t) - \hat{\mathrm{x}}(t)]$ is bounded as t goes to ∞).

Remark 1. Compared with the nonlinear estimation algorithms in Arasaratnam and Haykin (2009), Duan and Li (2015), Gordon et al. (1993), Julier and Uhlmann (1997, 2004), Lan and Li (2015), Psiaki et al. (2015) and Reif et al. (1999), the proposed nonlinear estimation method in this paper does not require any statistical information of noises. Moreover, the upper and lower bounds of the bounded noises in this paper are not required to be known. Since stochastic disturbances are always bounded in a practical system, the nonlinear estimation method in this paper is also applicable to the case of stochastic noises for a practical system (e.g., noises with truncated Gaussian distribution).

Remark 2. When designing the set-membership nonlinear estimation algorithms in Calafiore (2005) and Yang and Li (2010), the noises were assumed to be confined to specified ellipsoidal sets:

$$\mathrm{w}^{\mathrm{T}}(t)R_{\mathrm{w}}^{-1}(t)\mathrm{w}(t) \le 1, \ \mathrm{v}^{\mathrm{T}}(t)R_{\mathrm{v}}^{-1}(t)\mathrm{v}(t) \le 1 \tag{4}$$

where $R_{\mathrm{w}}(t) > 0$ and $R_{\mathrm{v}}(t) > 0$ were required to be known in advance, and the design of nonlinear estimators in Calafiore (2005) and Yang and Li (2010) was dependent on the matrices $R_{\mathrm{w}}(t)$ and $R_{\mathrm{v}}(t)$. However, the developed nonlinear estimation algorithm in this paper does not require to know the bounds of noises (see (2)). Meanwhile, the linearization errors and fuzzy errors in Calafiore (2005) and Yang and Li (2010) were modeled by parameter uncertainties with known lower and upper bounds, which may affect the applicability of set-membership nonlinear estimation algorithms. In contrast, the linearization error in this paper will be viewed as a bounded disturbance at a particular time. Then, the proposed