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Brief paper

Frequency domain identification of continuous-time output-error models with time-delay from relay feedback tests[☆]

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ABSTRACT

This paper is concerned with identification of continuous-time output-error models with time-delay from relay feedback tests. Conventional methods for solving this problem consist in deriving analytical limit cycle expressions and fitting them to measured shape factors. However, they may fail to handle different limit cycles uniformly, due to the structural differences in the analytical expressions. To overcome this problem, we consider a more general, data-based, parametric identification framework using sampled limit cycle data. A frequency domain method that minimizes the sum of squared output-errors is developed. The proposed method can be of high accuracy, thanks to the periodic input–output signals provided by sustained relay feedback oscillations, which can help to reduce leakage and aliasing errors. Besides, a distinctive merit of the proposed method is that identification of stable and unstable plants can be equally simple. The effectiveness and superiority of the proposed method are demonstrated by means of both theoretical analyses and simulation examples.

1. Introduction

Relay feedback tests were initially proposed in Åström and Hägglund (1984) for automatic tuning of PID controllers. The basic idea behind is to estimate the critical frequency and the corresponding plant gain from a relay-produced limit cycle via the so-called describing function (DP) method. Once the two shape factors have been obtained, the controller parameters can be readily determined. The main disadvantage of the DP method lies in an approximation of the system response by its first harmonic, which is valid only if the limit cycle is sufficiently near a sinusoid. For some circumstances such as high order or long time-delay processes, erroneous results may be generated (Atherton, 2006; Hang, Åström, & Wang, 2002). To circumvent this problem, an alternative is to apply model-based controller design instead of relay feedback auto-tuning. In particular, we first identify a process model from measured shape factors by using some system identification method, and subsequently we design controller parameters based on this identified model. Various methods for the identification purpose have been proposed in the literature. In Wang, Hang, and Zou (1997), exact expressions for periods and amplitudes of limit cycles were derived. On this basis, a method was proposed to identify first-order models plus dead-time from measured shape factors. For non-minimum phase systems, Majhi (2007) reported a method that is able to identify process models with four parameters at most. To handle the measurement noise at the system output, a curve fitting method was used to recover noise-free limit cycles.

The aforementioned methods have been successful in some process control applications. However, as it can be seen from Atherton (2006) and Hang et al. (2002), conventional methods may have the following two shortcomings:

(1) Limited flexibility of handling different limit cycles. As discussed in Majhi and Atherton (2000), the maximum number of parameters allowed to be estimated is dependent on the shape properties of a limit cycle. For example, from an odd symmetrical limit cycle two unknown parameters can be found, while from an asymmetrical limit cycle four can be found. In addition, the analytical expressions commonly differ from limit cycle to limit cycle (Panda & Yu, 2003) and, as a consequence, the solver for each limit cycle is more or

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less *ad hoc*. Therefore, there is a need to derive a method that is able to handle different limit cycles in a unified way.

(2) Sensitivity to measurement noise. In conventional methods, the unknown parameters are estimated by solving a set of algebraic equations constructed based on measured shape factors. Therefore, the parameter estimates can be accurate only if the limit cycles are noise-free. However, this is practically impossible due to inevitable disturbing noise on the system output and measurement. Thus, in a first step one needs to recover the noise-free limit cycles from their noisy observations. Without this, the optimality of parameter estimates, or even the existence of solutions, can be influenced. Several methods to this purpose have been reported in the literature, such as the curve fitting technique (Majhi, 2007). To the best of our knowledge, there is no easy way to access noise-free signals from their noisy observations, especially in the presence of a high noise level. Therefore, it is necessary to develop a method that is able to handle noisy data directly, as well as to generate consistent parameter estimates.

The complexity in shortcoming (1) stems from the use of analytical expressions which are highly nonlinear in parameters. Motivated by the fact that real-world systems to be identified are naturally continuous in time, it would be beneficial to consider a direct continuous-time (CT) identification approach, which parametrizes the plant by a CT transfer function in general form and then matches it to sampled input–output data via a least-squares criterion (Garnier, Mensler, & Richard, 2003; Garnier & Wang, 2008; Garnier & Young, 2014). It has the advantages that the input–output data and their derivatives are related to the parameters more conveniently than the limit cycle expressions are, and that the loss function to be minimized is quadratic. Also, direct CT identification does not require noise-free limit cycle data, as such the difficulty in shortcoming (2) can be avoided.

An attractive feature of relay feedback tests is the capability of handling unstable plants, thanks to the feedback mechanism that is able to prevent the process response from drifting too far. However, most data-based, time domain methods, for example Baysse, Carrillo, and Habbadi (2011), Chen, Garnier, and Gilson (2015), Chen, Gilson, Garnier, and Liu (2017), Chen, Zhuan, Garnier, and Gilson (2018), Ding, Wang, Mao, and Xu (2015), Gilson and Van den Hof (2005), Young (2015) and Young and Garnier (2006), cannot identify unstable plants directly, since they need stable predictors to compute gradients as well as to evaluate loss function values. A possibility is to identify the stable, closed-loop system with a controller as a whole, but this will make the identification more complicated. Fortunately, as pointed out in Pintelon and Schoukens (2012) and Pintelon, Schoukens, and Rolain (2008), there is no problem to model unstable plants in the frequency domain, since the transfer function is only computed on the imaginary axis. This motivates an extension of the time domain approach in our previous work (Chen et al., 2017) to the frequency domain so that both stable and unstable time-delay plants can be directly identified. For recent developments on frequency domain identification, the interested reader is directed to, *e.g.*, Gillberg and Ljung (2010), Gilson, Welsh, and Garnier (2018), Goos, Lataire, Louarroudi, and Pintelon (2017), Van den Hof and Douma (2008) and van Herpen, Oomen, and Steinbuch (2014) for more details. To sum up, the scope of this paper is to

(1) derive a frequency domain method following from the methods in Ljung (2002) and Pintelon and Schoukens (1997) towards direct identification of stable and unstable plants with time-delay from relay feedback tests;
(2) in the relay feedback framework, illustrate the merits of the proposed frequency domain identification method, and give a consistency analysis for the parameter estimates generated by the proposed method.

The remainder of this paper is organized as follows: The identification problem is formulated in Section 2. Subsequently, a frequency domain output-error (OE) method for time-delay CT plants is presented in Section 3. An analysis on the convergence of parameter estimates is demonstrated in Section 4. Thereafter, two numerical examples are presented in Section 5 to illustrate the effectiveness of the proposed method. Finally, conclusions are drawn in Section 6.

2. Problem formulation

Let us consider a linear time-invariant, single-input single-output, CT process with input $u(t)$ and noise-free response $x(t)$ related by the following differential equation of constant coefficients

$$x^{(n)}(t) + a_1^o x^{(n-1)}(t) + \cdots + a_n^o x(t) = b_0^o u^{(m)}(t-\tau_o) + \cdots + b_m^o u(t-\tau_o) \quad (1)$$

where $x^{(j)}(t)$ denotes the jth-order time-derivative of $x(t)$, $\tau_o \geqslant 0$ the pure time-delay, and $a_1^o, \ldots, a_n^o, b_0^o, \ldots, b_m^o$ the true process parameters. Note that no stability assumption is made and thus process (1) can either be stable or unstable.

2.1. Discrete-time measurements

In real-life, the measurement process is inevitably corrupted by noise. Even so, in some applications such as computer controlled systems, accessing the noise-free input is still possible since it is generated from a given sequence via a hold device. Therefore, it is reasonable to assume that the input is noise-free while the output, denoted by $z(t)$, is noise-corrupted

$$z(t) = x(t) + v(t). \quad (2)$$

Here, $v(t)$ is considered as CT white noise which cannot be predicted from past data. As illustrated in Ljung and Wills (2010), this leads to a mathematical difficulty that $v(t)$ has infinite variance. Thus, instantaneous samples of $z(t)$ cannot be obtained. It is possible that the state equation (1) is also affected by process noise. But this case is more difficult to tackle, see Ljung and Wills (2010). To remain simple, we assume that the system is only corrupted by measurement noise.

According to Åström (1970), CT white noise is a kind of signal that has a constant power spectral density (PSD) function over the frequency range $\omega \in (-\infty, \infty)$. Assuming that the PSD function of $v(t)$ is $S_v(\omega)$, the autocorrelation function of $v(t)$, denoted by $r_v(s)$, is linked to $S_v(\omega)$ by the well-known Wiener–Khinchin theorem

$$S_v(\omega) = \int_{-\infty}^{\infty} r_v(s) \mathrm{e}^{-\mathrm{i}\omega t} \mathrm{d}s \quad (3)$$

$$r_v(s) = \mathbb{E}\{v(t)v(t+s)\} \quad (4)$$

where $\mathrm{i}^2 = -1$ and $\mathbb{E}$ is the expectation operator. $S_v(\omega)$ is a real-valued function since $r_v(s)$ is symmetric. When $S_v(\omega) = \sigma^2$ is a constant, by using the inverse Fourier transform (FT), it can be shown that

$$r_v(s) = \frac{1}{2\pi} \int_{-\infty}^{\infty} S_v(\omega) \mathrm{e}^{\mathrm{i}\omega s} \mathrm{d}\omega = \sigma^2 \delta(s) \quad (5)$$

where $\delta(s)$ is Dirac's delta function. The above result explains why CT white noise has infinite variance.