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Brief paper

Adaptive asymptotic tracking using barrier functions☆

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ABSTRACT

This paper studies the global output tracking problem for a class of unknown time-varying nonlinear systems in strict-feedback form. By utilizing the barrier functions, a universal adaptive state-feedback control strategy is proposed that achieves asymptotic tracking performance. Unlike the existing results in the literature, the proposed control scheme utilizes the barrier functions to ensure the unknown system nonlinearities to be the bounded "disturbance-like" terms, which are adaptively compensated at each step, this enables any approximation structures are not needed. Furthermore, the "explosion of complexity" issue in backstepping-like approaches is avoided without using additional filtering. Simulation results are presented to demonstrate the effectiveness of the proposed methodology.

1. Introduction

During the past several decades, control problem of uncertain nonlinear systems has attracted considerable attentions. Significant strides, including adaptive feedback linearization (Sastry & Isidori, 1989), adaptive backstepping (Krstic, Kanellakopoulos, & Kokotovic, 1995), immersion and invariance adaptive control (Astolfi, Karagiannis, & Ortega, 2007), and sliding mode control (Utkin, 1992), have been proposed in the literature. However, in all aforementioned results, to propose the nonlinear controllers, a priori knowledge of system nonlinearities is needed to be known. For example, it is required that the system nonlinearities be known, or be bounded by known functions, or be linearly parameterized with unknown parameters, or satisfy some (high order) growth rate condition, etc.

In case of nonlinear systems with completely unknown dynamics, neural networks and fuzzy systems have been extensively utilized to design adaptive controllers (Farrell & Polycarpou, 2006; Ge, Hang, Lee, & Zhang, 2002; Lewis, Jagannathan, & Yesildirek, 1999; Sanner & Slotine, 1992; Spooner, Maggiore, Ordonez, & Passino, 2002; Wang, Lu, Gao, & Liu, 2017), owing to their universal approximation capacities. Nevertheless, it is worth noting that the approximation capacities of these universal approximators are just limited to a compact domain. Consequently, almost all adaptive neural/fuzzy control schemes only guarantee semi-global stability of the resulting closed loop system. Instead of using approximation structures, switching universal control approaches were proposed for global stabilization of unknown feedforward nonlinear systems (Ye, 1999, 2011; Ye & Unbehauen, 2004) and feedback nonlinear systems (Ma, Liu, Zhao, Wang, & Zong, 2015; Ye, 2003, 2005, 2012). The proposed switching universal control approaches can also be extended to the tracking problem. However, asymptotic tracking can no longer be achieved, even in the linear parametrization case (Ye, 2012). To achieve the output tracking with prescribed transient behavior, the funnel control approach was first developed in Ilchmann, Ryan, and Sangwin (2002) for systems with relative degree one, where a nonlinear and time-varying proportional controller was designed to force the output error approach the funnel boundary. Unfortunately, for systems with higher relative degree, the funnel controller using backstepping procedure is quite complicated and impractical, since it involves high powers of a gain function which typically takes very large values (Ilchmann, Ryan, & Townsend, 2007). As an alternative, Prescribed Performance Control (PPC), introduced in Bechlioulis and Rovithakis (2008, 2009, 2010), utilizes the appropriately defined functions to transform the original system into one that incorporates the desired performance

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specifications. Inspired by PPC, a low-complexity approximation-free universal control scheme was proposed for unknown pure-feedback systems (Bechlioulis & Rovithakis, 2014). To deal with the general dead-zone input nonlinearity, a PPC-based low-complexity controller was presented in Theodorakopoulos and Rovithakis (2015). By introducing novel error transformation functions and using a Nussbaum-type function, a universal PPC scheme was developed for unknown strict-feedback systems with actuator faults and unknown control directions (Zhang & Yang, 2017).

A common obstacle for the aforementioned universal control methodologies is that the asymptotic tracking performance cannot be guaranteed in the presence of unknown system nonlinearities. Instead, only the boundedness of the tracking error or adaptive regulation was established. A question naturally arises: *is it possible to design a global universal controller for unknown nonlinear systems to achieve asymptotic tracking?* To our best knowledge, in the existing literature there have not been results which provide answers to this question.

In this paper, we will concentrate on this problem and propose a universal adaptive control approach to realize the asymptotic tracking for a general class of unknown time-varying nonlinear systems. Compared to the relevant existing results in the literature, the main contributions of this paper are as follows.

- This work is the first to address the problem of global asymptotic tracking for nonlinear time-varying systems with unknown dynamics. Without any analytical expressions of system nonlinearities, a universal adaptive control approach is developed that achieves asymptotic tracking performance.
- Instead of using approximation structures, in the proposed approach, the unknown system nonlinearities are enforced to be the bounded "disturbance-like" terms via barrier Lyapunov functions, which are adaptively compensated at each step.
- Compared with backstepping-like approaches, the proposed universal control scheme does not incorporate derivatives of virtual control variables or desired signal, thus avoiding the problem of "explosion of complexity" without residing in filtering.

The rest of this paper is structured as follows. Section 2 introduces the system description and some preliminaries. The universal adaptive control approach is presented in Section 3. Section 4 is devoted to validate the proposed scheme through two illustrative examples. Some conclusions are drawn in Section 5.

2. Problem statement and preliminaries

Consider the following SISO strict-feedback nonlinear time-varying system:

$$\begin{aligned}\dot{x}_i &= f_i(t,\bar{x}_i)+g_i(t,\bar{x}_i)x_{i+1},\quad i=1,\ldots,n-1,\\ \dot{x}_n &= f_n(t,\bar{x}_n)+g_n(t,\bar{x}_n)u,\\ y &= x_1\end{aligned}\tag{1}$$

where $\bar{x}_i=[x_1,\ldots,x_i]^T\in R^i$, $i=1,\ldots,n$; $\bar{x}_n\in R^n$ are the states with initial conditions $\bar{x}_n^0=[x_1^0,\ldots,x_n^0]^T=[x_1(0),\ldots,x_n(0)]^T$, $u\in R$ is the control input, $y\in R$ is the output; The system nonlinearities $f_i, g_i: R_+\times R^i\to R$, $i=1,\ldots,n$ are piecewise continuous in t and locally Lipschitz in $\bar{x}_i$. The control objective of this paper is to design a universal controller u such that the system output asymptotically converges to a desired trajectory y_r while ensuring that all signals in the closed loop system are bounded.

To facilitate the control design, the following assumptions are made.

Assumption 1. The functions g_i, $i=1,\ldots,n$ are either strictly positive or strictly negative. Further, their signs are considered known.

Assumption 2. There exist continuous non-negative functions $\bar{f}_i(\bar{x}_i)$, $i=1,\ldots,n$ such that

$$|f_i(t,\bar{x}_i)|\le\bar{f}_i(\bar{x}_i),\quad i=1,\ldots,n.\tag{2}$$

Further, there exist continuous and strictly positive functions $\bar{g}_i(\bar{x}_i)$ and $\underline{g}_i(\bar{x}_i)$, $i=1,\ldots,n$ such that

$$\underline{g}_i(\bar{x}_i)\le|g_i(t,\bar{x}_i)|\le\bar{g}_i(\bar{x}_i),\quad i=1,\ldots,n.\tag{3}$$

Assumption 3. The desired trajectory y_r is bounded, continuous and available, and $\dot{y}_r$ is bounded but may be not available.

Remark 1. Assumption 1 is a sufficient condition for global controllability of system (1). Assumption 2 implies that functions $|f_i|$, $|g_i|$, $i=1,\ldots,n$ may not grow arbitrarily large and $|g_i|$ may not approach zero owing to the variation of t, which ensures boundedness of the virtual and actual control signals. Moreover, as mentioned in Assumption 3, only y_r and none of its high order derivatives is available for control design, in contrast to backstepping-like procedures (Farrell, Polycarpou, Sharma, & Dong, 2009; Krstic et al., 1995; Liu, 2018; Swaroop, Hedrick, Yip, & Gerdes, 2000) which require the availability of the aforementioned derivatives.

In the following, we list two lemmas which play an important role in deriving a solution to the universal asymptotic tracking problem for system (1).

Lemma 1 (*Zuo & Wang, 2014*)**.** *The following inequality holds for any $\epsilon>0$ and for any $z\in R$:*

$$0\le|z|-\frac{z^2}{\sqrt{z^2+\epsilon^2}}<\epsilon.\tag{4}$$

Lemma 2 (*Ren, Ge, Tee, & Lee, 2010*)**.** *The following inequality holds for any $z\in R$ satisfying $|z|<1$:*

$$\log\frac{1}{1-z^2}\le\frac{z^2}{1-z^2}.\tag{5}$$

3. Universal adaptive control design

In this section, a universal adaptive control approach will be proposed by using barrier Lyapunov functions (the readers may refer to Tee, Ge, and Tay (2009) for more details about barrier Lyapunov functions). Before presenting the proposed universal controller, let us define the constrained function $k(t)$ as follows.

Definition 1. A continuous function $k(t): R_+\to R_+-\{0\}$ is said to be a constrained function if $\underline{k}\le k(t)\le\bar{k}$, $|\dot{k}(t)|\le\kappa$, where $\underline{k}$, $\bar{k}$ and κ are positive constants.

3.1. Universal adaptive control scheme

As usual in backstepping approach, the following change of coordinates is made:

$$z_1=x_1-y_r-\varrho(t)(x_1^0-y_r(0))\tag{6}$$

$$z_i=x_i-\alpha_{i-1}-\varrho(t)x_i^0,\quad i=2,\ldots,n\tag{7}$$