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Brief paper

An improved time-delay implementation of derivative-dependent feedback[☆]

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ABSTRACT

We consider an LTI system of relative degree $r \geq 2$ that can be stabilized using $r-1$ output derivatives. The derivatives are approximated by finite differences leading to a time-delayed feedback. We present a new method of designing and analyzing such feedback under continuous-time and sampled measurements. This method admits essentially larger time-delay/sampling period compared to the existing results and, for the first time, allows to use consecutively sampled measurements in the sampled-data case. The main idea is to present the difference between the derivative and its approximation in a convenient integral form. The kernel of this integral is hard to express explicitly but we show that it satisfies certain properties. These properties are employed to construct the Lyapunov–Krasovskii functional that leads to LMI-based stability conditions. If the derivative-dependent control exponentially stabilizes the system, then its time-delayed approximation stabilizes the system with the same decay rate provided the time-delay (for continuous-time measurements) or the sampling period (for sampled measurements) are small enough.

1. Introduction

Control laws that depend on output derivatives are used to stabilize LTI systems with relative degrees greater than one. To estimate the derivatives, which can hardly be measured directly, one can use the finite differences, i.e., $\dot{y} \approx (y(t)-y(t-h))/h$. Such approximation leads to time-delayed feedback that preserves the stability if the delay $h > 0$ is small enough (Borne, Kolmanovskii, & Shaikhet, 2000; French, Ilchmann, & Mueller, 2009; Karafyllis, 2008). For a given h, the delay-induced stability can be checked using frequency-domain techniques (Abdallab, Dorato, & Benites-Read, 1993; Kharitonov, Niculescu, Moreno, & Michiels, 2005; Niculescu & Michiels, 2004; Ramírez, Mondié, Garrido, & Sipahi, 2016) or complete Lyapunov–Krasovskii functionals (Egorov, 2016; Gu, Kharitonov, & Chen, 2003; Kharitonov, 2012), which give necessary and sufficient conditions.

The delay-induced stability can be also studied using linear matrix inequalities (LMIs) (Gu, 1997; Seuret & Gouaisbaut, 2013, 2015). The advantage of LMIs is that, though being conservative, they allow for performance and robustness analysis, can cope with certain types of nonlinearities (Fridman, 2014), and can deal with stochastic perturbations (Fridman & Shaikhet, 2016, 2017). Simple and yet efficient LMIs for the delay-induced stability were obtained in Fridman and Shaikhet (2016, 2017). The key idea was to use Taylor's expansion of the delayed terms with the remainders in the integral form that are compensated by appropriate terms in the Lyapunov–Krasovskii functional. Compared to Gu (1997) and Seuret and Gouaisbaut (2013, 2015), the resulting LMIs have a lower order, contain less decision variables, and were proved to be feasible for small delays if the derivative-dependent feedback stabilizes the system.

Another advantage of LMI-based conditions is that they can be extended to sampled-data systems. This has been done using discretized Lyapunov functionals with a Wirtinger-based term in Liu and Fridman (2012). Another LMIs for sampled-data stabilization were derived in Seuret and Briat (2015) by employing impulsive system representation and looped Lyapunov functionals. The high-order LMIs obtained in Liu and Fridman (2012) and Seuret and Briat (2015) contain many decision variables, which make them hard to solve numerically. Using the ideas of Fridman and Shaikhet (2016, 2017), simple LMIs for sampled-data delay-induced stabilization were derived in Selivanov and Fridman (in press-b). These conditions were proved to be feasible for a small enough sampling period if the continuous-time derivative-dependent feedback stabilizes the system.

In this paper, we essentially improve the results of Fridman and Shaikhet (2017) for continuous-time measurements (Section 2)

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and the results of Selivanov and Fridman (in press-b) for sampled measurements (Section 3). Namely, we derive simple LMIs that are feasible for significantly larger values of time-delay (Remark 2) and sampling period (Remark 3). Such improvement is achieved using an original integral representation of the difference between the derivative and its approximation (Proposition 1). The kernel of this integral is hard to express explicitly but we show that it satisfies certain properties (Proposition 2). These properties are employed to construct Lyapunov–Krasovskii terms that bound the approximation errors and lead to LMI-based stability conditions. Compared to Fridman and Shaikhet (2017) and Selivanov and Fridman (in press-b), such approach leads to a more natural design of the controller gains in the delayed feedback. Moreover, the considered sampled-data delayed controller uses *consecutive* measurements, while Selivanov and Fridman (in press-b) used *distant* measurements (cf. (25) and (29)). All these improvements allow to use less memory and slower sampling when one uses time-delays to implement derivative-dependent feedback. Finally, we show that if the derivative-dependent controller exponentially stabilizes the system with a decay rate $\alpha' > 0$, then the LMIs are feasible for any decay rate $\alpha < \alpha'$ and small enough time-delay/sampling period.

The part of this paper corresponding to the sampled-data implementation of the first order derivative was presented in Selivanov and Fridman (2018). These results were used in Selivanov and Fridman (in press-a) to study sampled-data implementation of PID control.

Notations. $\mathbb{N}_0 = \mathbb{N} \cup \{0\}$, $\mathbf{1}_r = [1, \dots, 1]^T \in \mathbb{R}^r$, $I_m \in \mathbb{R}^{m\times m}$ is the identity matrix, $\otimes$ stands for the Kronecker product, $\mathrm{diag}\{R_i\}_{i=1}^{r-1}$ is the block-diagonal matrix with R_i being on the diagonal, $0 < P \in \mathbb{R}^{n\times n}$ denotes that P is symmetric and positive-definite, C^i is a class of i times continuously differentiable functions.

Auxiliary lemmas.

Lemma 1 (*Exponential Wirtinger Inequality, Selivanov & Fridman, 2016*). *Let $f : [a, b] \to \mathbb{R}^n$ be an absolutely continuous function with a square integrable first order derivative such that $f(a) = 0$ or $f(b) = 0$. Then*

$$\int_a^b e^{2\alpha t} f^T(t) W f(t)\, dt \le e^{2|\alpha|(b-a)} \frac{4(b-a)^2}{\pi^2} \int_a^b e^{2\alpha t} \dot f^T(t) W \dot f(t)\, dt$$

for any $\alpha \in \mathbb{R}$ and $0 \le W \in \mathbb{R}^{n\times n}$.

Lemma 2 (*Jensen's Inequality, Solomon & Fridman, 2013*). *Let $\rho : [a, b] \to [0, \infty)$ and $f : [a, b] \to \mathbb{R}^n$ be such that the integration concerned is well-defined. Then for any $0 < Q \in \mathbb{R}^{n\times n}$,*

$$\left[\int_a^b \rho(s) f(s)\, ds\right]^T Q \left[\int_a^b \rho(s) f(s)\, ds\right] \le \int_a^b \rho(s)\, ds \int_a^b \rho(s) f^T(s) Q f(s)\, ds.$$

2. Continuous-time control

Consider the LTI system

$$\begin{aligned} \dot x(t) &= Ax(t) + Bu(t), \\ y(t) &= Cx(t), \end{aligned} \qquad x \in \mathbb{R}^n, u \in \mathbb{R}^m, y \in \mathbb{R}^l \tag{1}$$

with relative degree $r \ge 2$, i.e.,

$$CA^iB = 0, \quad i = 0, 1, \dots, r-2, \qquad CA^{r-1}B \ne 0. \tag{2}$$

Relative degree is how many times the output $y(t)$ needs to be differentiated before the input $u(t)$ appears explicitly. In particular, (2) implies

$$y^{(i)} = CA^i x, \quad i = 0, 1, \dots, r-1. \tag{3}$$

To prove (3), note that it is trivial for $i = 0$ and, if it has been proved for $i < r-1$, it holds for $i+1$:

$$y^{(i+1)} = \left(y^{(i)}\right)' \overset{(3)}{=} (CA^i x)' \overset{(1)}{=} CA^i[Ax + Bu] \overset{(2)}{=} CA^{i+1}x.$$

For LTI systems with relative degree r, it is common to look for a stabilizing controller of the form

$$u(t) = \bar K_0 y(t) + \bar K_1 y^{(1)}(t) + \cdots + \bar K_{r-1} y^{(r-1)}(t). \tag{4}$$

Remark 1. The control law (4) essentially reduces the system's relative degree from $r \ge 2$ to $r = 1$. Indeed, due to (2), the transfer matrix of (1) has the form

$$W(s) = \frac{\beta_r s^{n-r} + \cdots + \beta_n}{s^n + \alpha_1 s^{n-1} + \cdots + \alpha_n}$$

with $\beta_r = CA^{r-1}B \ne 0$. Taking $u(t) = \hat K_0 u_0(t) + \hat K_1 u_0^{(1)}(t) + \cdots + \hat K_{r-1} u_0^{(r-1)}(t)$, one has

$$\tilde y(s) = \frac{(\beta_r s^{n-r} + \cdots + \beta_n)(\hat K_{r-1} s^{r-1} + \cdots + \hat K_0)}{s^n + \alpha_1 s^{n-1} + \cdots + \alpha_n} \tilde u_0(s),$$

where $\tilde y$ and $\tilde u_0$ are the Laplace transforms of y and u_0. If $\beta_r \hat K_{r-1} \ne 0$, the latter system has relative degree one. If it can be stabilized by $u_0 = Ky$ then (1) can be stabilized by (4) with $\bar K_i = \hat K_i K$.

The controller (4) depends on the output derivatives, which are hard to measure directly. Instead, the derivatives can be approximated by finite-differences $\tilde y_i(t) \approx y^{(i)}(t)$:

$$\begin{aligned} \tilde y_0(t) &= y(t), \\ \tilde y_i(t) &= \frac{\tilde y_{i-1}(t) - \tilde y_{i-1}(t-h)}{h} \\ &= \frac{1}{h^i} \sum_{k=0}^{i} \binom{i}{k} (-1)^k y(t-kh), \quad i \in \mathbb{N} \end{aligned} \tag{5}$$

with a delay $h > 0$ and the binomial coefficients $\binom{i}{k} = \frac{i!}{k!(i-k)!}$. Replacing $y^{(i)}$ in (4) with their approximations $\tilde y_i$, we obtain the delay-dependent control

$$u(t) = \sum_{i=0}^{r-1} \bar K_i \tilde y_i(t) \overset{(5)}{=} \sum_{i=0}^{r-1} K_i y(t - ih), \tag{6}$$

where we set[1] $y(t) = y(0)$ for $t < 0$ and

$$K_i = (-1)^i \sum_{j=i}^{r-1} \binom{j}{i} \frac{1}{h^j} \bar K_j, \quad i = 0, \dots, r-1. \tag{7}$$

If (1) can be stabilized by the derivative-dependent control (4), then it can be stabilized by the delay-dependent control (6) with small enough delays (French et al., 2009). In this section, we derive simple and yet efficient LMIs that allow to obtain appropriate value of the delay $h > 0$. The first step is to present the approximation error $y^{(i)}(t) - \tilde y_i(t)$ in a convenient form suitable for the analysis via Lyapunov–Krasovskii functionals.

Proposition 1. *If $y \in C^i$ and $y^{(i)}$ is absolutely continuous with $i \in \mathbb{N}$, then $\tilde y_i$ defined in (5) satisfies*

$$\tilde y_i(t) = y^{(i)}(t) - \int_{t-ih}^{t} \varphi_i(t-s) y^{(i+1)}(s)\, ds, \tag{8}$$

[1] Then $y^{(i)}(0)$ with $i > 0$ are approximated by 0.