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A note on Parisian ruin under a hybrid observation scheme

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ABSTRACT

In this paper, we study the concept of Parisian ruin under the hybrid observation scheme model introduced by Li et al. (2018). Under this model, the process is observed at Poisson arrival times whenever the business is financially healthy and it is continuously observed when it goes below 0. The Parisian ruin is then declared when the process stays below zero for a consecutive period of time greater than a fixed delay. We improve the result originally obtained in Li et al. (2018) and we compute other fluctuation identities. All identities are given in terms of second-generation scale functions.

1. Introduction

Parisian ruin is a relaxation of the classical ruin since a delay is allowed before ruin occurs. More precisely, Parisian ruin occurs if the time spent below a pre-determined critical level for a consecutive period is greater than a pre-specified delay. Two types of Parisian ruin have been considered, one with fixed delays and another one with stochastic delays. Lkabous and Renaud (2018a) unified these two types of Parisian ruin into one called *mixed Parisian ruin*. In this case, ruin is declared the first time an excursion into the red zone lasts longer than an implementation delay with a deterministic and a stochastic component. Recently, more definitions of Parisian ruin have been proposed. Cumulative Parisian ruin has been proposed in Guérin and Renaud and Lkabous and Renaud (2018b); in that case, the *race* is between a single deterministic clock and the sum of the excursions below the critical level. Moreover, in Czarna and Renaud (2016), Parisian ruin with an ultimate bankruptcy level for Lévy insurance risk processes was considered. This type of ruin occurs if either the process goes below a predetermined negative level or if Parisian ruin with deterministic delays occurs.

Recently, Poisson observations problems have attracted considerable attention. In this case, the risk process is monitored discretely at arrival epochs of an independent Poisson process, which can be interpreted as the observation times of the regulatory body, see Albrecher et al. (2016) and Li et al. (2018) among others. A new definition of ruin has been studied in a spectrally negative Lévy setup by Li et al. (2018). They introduced the idea of Parisian ruin under a hybrid observation scheme. More specifically, when the risk process is above 0, it is monitored discretely at Poisson arrival times until a negative surplus is observed. Then, the process will be observed continuously and a grace period is granted for the insurance company to recover to a solvable level $a \geq 0$. Otherwise, Parisian ruin occurs.

In this paper, we study a Parisian ruin under a hybrid observation scheme for a general Lévy insurance risk model as defined in Li et al. (2018). We present a probabilistic analysis and simple resulting expressions for the two-sided exit problem, Laplace transform and the probability of Parisian ruin under a hybrid observation scheme in terms of the delayed scale functions introduced by Lkabous and Renaud (Lkabous and Renaud, 2018a). Our approach is based on the expression of

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the Gerber–Shiu distribution at Parisian ruin with exponential implementation delays obtained in Baurdoux et al. (2016), combined with some results in Lkabous et al. (2017).

The rest of the paper is organized as follows. In Section 2, we present the necessary background material on spectrally negative Lévy processes and classical scale functions, including delayed scale functions and some fluctuation identities with delays already available in the literature. The main results are presented in Section 3, followed by a discussion on those results. In Section 4, we derive new technical identities and then provide proofs for the main results. In the last section, we provide explicit computations of the probability of Parisian ruin under a hybrid observation scheme for Brownian risk model.

1.1. Parisian ruin under a hybrid observation scheme

First, we denote the first passage times of X above b

$$\tau_b^+ = \inf\{t \geq 0 : \ X_t \geq b\}.$$

For a standard Lévy insurance risk process X, the time of Parisian ruin, with delay $r > 0$, has been studied in Loeffen et al. (2013): it is defined as

$$\kappa_r = \inf\{t > 0 : t - g_t > r\}, \tag{1}$$

where $g_t = \sup\{0 \leq s \leq t : X_s \geq 0\}$. Then, Parisian ruin occurs the first time an excursion below zero lasts longer than the fixed implementation delay r. Loeffen et al. (2013) obtained a very nice and compact expression for the probability of Parisian ruin : for $r > 0$ and $x \in \mathbb{R}$, we have

$$\mathbb{P}_x(\kappa_r < \infty) = 1 - (\mathbb{E}[X_1])_+ \frac{\int_0^\infty W(x+z) z \mathbb{P}(X_r \in \mathrm{d}z)}{\int_0^\infty z \mathbb{P}(X_r \in \mathrm{d}z)}, \tag{2}$$

where $(x)_+ = \max(x, 0)$ and W is the so-called scale function of X (see the definition in the next section).

Parisian ruin with stochastic delay has also been considered in Baurdoux et al. (2016); Landriault et al. (2011, 2014). In this case, the fixed delay r is replaced by an independent exponential random and it occurs the first time when the length an excursion below 0 is longer than the exponential clock. It also corresponds to the first passage time when X is observed at Poisson arrival times below 0, that is

$$T_0^- = \min\{T_i > 0 : X_{T_i} < 0, \ i \in \mathbb{N}\}, \tag{3}$$

where T_i are the arrival times of an independent Poisson process of rate $\lambda > 0$. We will use the notation T_0^- instead of κ_λ.

In Li et al. (2018), the time of Parisian ruin under a hybrid observation scheme with recover barrier $a \geq 0$ and a fixed delay $r > 0$ is defined as

$$\tilde{\kappa}_{a,r}^{\lambda} = \inf\left\{t \in \left(T_n, \tau_a^+ \circ \theta_{T_n}\right) : X_{T_n} < 0 \text{ and } t - T_n \geq r, n \in \mathbb{N}\right\},$$

where θ is the Markov shift operator ($X_s \circ \theta_t = X_{s+t}$). In other words, when the risk process is above the level $a \geq 0$, it is monitored discretely at Poisson arrival times until a negative surplus is observed. Then, the process will be observed continuously and a fixed delay r is granted for the insurance company to recover to the solvable level a.

Li et al. (2018) obtained the following expression for the probability of Parisian ruin under a hybrid observation scheme using a standard probabilistic decomposition and using the technique of taking Laplace transform with respect to the delay r.

Theorem 1. *For $r, \lambda > 0$, $a \geq 0$ and $x \in \mathbb{R}$, we have*

$$\mathbb{P}_x\left(\tilde{\kappa}_{a,r}^{\lambda} < \infty\right) = 1 - \psi'(0+) \frac{\Phi_\lambda}{\lambda} Z(x, \Phi_\lambda) - \psi'(0+) \frac{\Phi_\lambda}{\lambda} \frac{Z(a, \Phi_\lambda)\, \lambda \int_0^r \mathrm{e}^{\lambda(r-s)} g_{x,a,\lambda}(s)\, \mathrm{d}s}{1 - \lambda \int_0^r \mathrm{e}^{\lambda(r-s)} g_{a,a,\lambda}(s)\, \mathrm{d}s},$$

where

$$g_{x,a,\lambda}(s) = \int_a^\infty \left(\frac{\Phi_\lambda}{\lambda} Z(x, \Phi_\lambda) - W(x+z-a)\right) \frac{z}{r} \mathbb{P}(X_r \in \mathrm{d}z).$$

We want to improve on this result by making it more close to Eq. (2) using a probabilistic approach. Without loss of generality, we will assume that $a = 0$ and we will write $\tilde{\kappa}_{0,r}^{\lambda} = \tilde{\kappa}_r^{\lambda}$.

2. Lévy insurance risk processes

We say that $X = \{X_t, t \geq 0\}$ is a Lévy insurance risk process if it is a spectrally negative Lévy process (SNLP) on the filtered probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t, t \geq 0\}, \mathbb{P})$, that is a process with stationary and independent increments and no positive jumps. To avoid trivialities, we exclude the case where X has monotone paths.