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On total positivity of exchangeable random variables obtained by symmetrization, with applications to failure-dependent lifetimes

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Abstract

Necessary and sufficient conditions for multivariate total positivity of order 2 (MTP$_2$) for density functions of some class of exchangeable random variables are obtained. The considered densities occur via symmetrization of particular ordered random variables. As an example, a characterization of the MTP$_2$ property for the Freund–Weinman multivariate exponential distribution is given. Furthermore, the results are applied to general failure-dependent component lifetimes in systems based on sequential order statistics. In the latter setting, the hazard rate increasing upon failure (HIF) property is also characterized. In particular, the case of underlying distributions satisfying the proportional hazards assumption is considered. The results are supplemented by an analysis of the covariances of the above multivariate exponential distribution.

Keywords: Exchangeable random variables, Freund–Weinman multivariate exponential distribution, Hazard rate increasing upon failure, Multivariate total positivity, Sequential order statistics, Symmetrized density function
2010 MSC: Primary 60E15, Secondary 62H05, 62N05

1. Introduction

Models of exchangeable random variables often provide natural extensions when the independence assumption in a classical setting of iid random variables should be relaxed. The focus of this paper lies on studying dependence properties of exchangeable random variables that arise in a natural way via symmetrization of particular ordered random variables. In general, the concept of symmetrization refers to the distribution of an exchangeable random vector $(Y_1, \ldots, Y_n)$ which is obtained as the random permutation of the entries of an arbitrary random vector $(X_1, \ldots, X_n)$, i.e.,

$$(Y_1, \ldots, Y_n) =_d (X_{R_1}, \ldots, X_{R_n})$$

where the random vector $(R_1, \ldots, R_n)$ is uniformly distributed on the set of permutations of $\{1, \ldots, n\}$. Note that the random vectors $(X_1, \ldots, X_n)$ and $(R_1, \ldots, R_n)$ are assumed to be independent. The concept of symmetrization can be used for constructing exchangeable random variables whose order statistics possess the same joint distribution as order statistics from given (possibly non-exchangeable) random variables; see, e.g., [28, 35, 36].

The main result of our paper is a characterization of the MTP$_2$ property for random vectors with a specific type of permutation invariant density function that matches the above setting. Recall that MTP$_2$ is known as one of the strongest notions of positive dependence; see, e.g., [26, 31, 35]. Our study is motivated by a model of exchangeable lifetimes from reliability theory. A particular case of the density structure considered in our main result arises by symmetrizing the joint density of sequential order statistics of Kamps [18]; see also [9, 10]. The corresponding exchangeable random variables can be interpreted as failure-dependent component lifetimes in systems based on sequential order statistics; see [1, 7, 27]. The joint distribution of these component lifetimes represents a dependence model which is capable of describing the behavior of component lifetimes in coherent systems, where failures may

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