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Heat transfer increase for a laminar pipe flow of a magnetic fluid subjected to constant heat flux: Improvements and additional discussions

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ABSTRACT

This work presents important additional discussions regarding our recent publication: *Heat transfer increase for a laminar pipe flow of a magnetic fluid subjected to constant heat flux: an initial theoretical approach.* In our first paper we proposed a theoretical law to predict the mean Nusselt number for a laminar pipe flow of a magnetic fluid subjected to a constant heat flux from the walls. We have considered a magnetic fluid flowing inside a small pipe under the action of an uniform magnetic field. From our analysis we have proposed a functional dependency on the mean Nusselt number with respect to magnetic parameters. This increase/decrease in the heat transfer rates inside the pipe arose from a production term in the equation of energy. We have interpreted it as a mechanism related to the deformation of clusters of magnetic particles formed in the microstructure of the fluid. In the present manuscript we present additional physical discussions regarding our previous derivation and propose a correction parameter ζ in this mechanism. This parameter intends to provide a more realistic modeling of this phenomenon, consistent with the restrictive assumptions assumed on our first paper. We also present some new results using Langevin Dynamics to show that this production term spontaneously appear when a non-superparamagnetic fluid is subjected to a shear field under a constant uniform magnetic field.

1. Introduction

In our previous paper *Heat transfer increase for a laminar pipe flow of a magnetic fluid subjected to constant heat flux: an initial theoretical approach* [1] we proposed a theoretical law to compute the mean Nusselt number (Nu_D) for a superparamagnetic fluid pipe flow subjected to both constant heat flux and magnetic field. The idea of this paper was to show from the balance equations of fluid mechanics a functional dependency of the heat transfer rates for this flow with respect to magnetic variables. On the past, several authors have explored the possibility of controlling the convective heat transfer rates of specific flows using magnetic fluids [2–8]. This idea motivated us to search for a functional dependency between Nu_D and practical operational parameters based on magnetic variables. We have then defined a set of very restrictive physical assumptions in order to isolate a possible mechanism that could lead us to this functional dependency without the necessity of performing experiments or numerical simulations. We have assumed in our first paper the following restrictive assumptions: (1) fully (hydrodydynamic and thermal) developed pipe flow; (2) superparamagnetism assumption: magnetic particles respond instantaneously to the action of an external field; (3) the fluid is subjected to an uniform applied field.

From a purely mathematical perspective, these assumptions lead us to the elimination of a hydrodydynamic-thermal-magnetic coupling. The hypothesis of superparamagnetism leads to a single magnetic term in the equation of motion, which is the vastly known *Kelvin force* [9–11]. On the other hand, the assumption of a constant applied field $\boldsymbol{H_0}$ eliminates any magnetic effect in the equations of motion. However, the term $\boldsymbol{\sigma}: \nabla \boldsymbol{v}$ (being $\boldsymbol{\sigma}$ the stress tensor of the medium and $\boldsymbol{v}$ the velocity flow field) which appears explicitly in the equation of energy could still be non-null in this scenario considering the known constitutive equations for the Maxwell stress tensor in the superparamagnetic limit. This leads to a possible coupling between magnetic and thermal effects. Despite the correct calculations of these magnetic effects in this particular flow, it is necessary to justify from a physical perspective the origin of this production term on the equation of energy. We could speculate the existence of an internal friction mechanism responsi-

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ble for the term $\boldsymbol{\sigma}^{\boldsymbol{M}}\colon \nabla \boldsymbol{v}$ (where $\boldsymbol{\sigma}^{\boldsymbol{M}}$ represents magnetic stresses), but this mechanism would alter the pressure loss inside the pipe and hence would not be captured by the joint assumptions of paralelism between the fluid's magnetization and the uniform applied field.

Still, we were able to propose a functional dependency on the mean Nusselt number of this particular problem with respect to important operational variables, such as: the intensity of the applied field H_0, the fluid's magnetic susceptibility χ, the orientation angle θ between the applied field and the flow, the heat flux on the wall q_s'', the tube's diameter D and the flow rate Q. Our last proposal was as a first approach of this problem. Moreover, we have identified a physical dimensionless parameter $\lambda_m = f(H_0, \chi, q_s'', D, Q)$ which relates the magnetic effects arising from the term $\boldsymbol{\sigma}^{\boldsymbol{M}}\colon \nabla \boldsymbol{v}$ (in the energy equation) with Nu_D and a function of θ that estimates the optimal orientation angle to achieve a desired increase/decrease on the heat transfer rates of the flow. The idea is that this functional dependency should be multiplied by a calibration parameter ζ in order to justify the source of this production term, which has been neglected up to now due to the restrictive assumptions assumed in the first paper. Moreover, we would like to discuss the functional dependency of this parameter using numerical simulation results in the framework of Langevin Dynamics [12,13].

Now we present additional discussions and comments regarding our last paper, propose a correction parameter ζ for the expression obtained in our previous work and use some new numerical simulation results based on Langevin Dynamics in order to avail the magnitude and functional dependencies of the term $\boldsymbol{\sigma}^{\boldsymbol{M}}\colon \nabla \boldsymbol{v}$ for a simple flow field (simple shear) acting under a continuum volume filled with magnetic nanoparticles subjected to a constant applied field.

2. Previous deduction

Here we will not repeat the deduction presented in our latest paper [1]. We will only highlight the most controversial points in order to draw the attention of the reader to the discussions presented in the next section. For the details of our previous deductions, please consult [1].

In our first paper [1], our start point was the set of balance equations (mass conservation, balance of linear momentum and energy) for a continuum medium in the framework of an Eulerian formalism. In these equations, we must define a constitutive law to model the state of stresses within the medium, $\boldsymbol{\sigma}$. We have assumed the medium as a superparamagnetic liquid. The state of stresses inside the medium were modeled by decomposing the Newtonian, $\boldsymbol{\sigma}^{\boldsymbol{N}}$, and magnetic, $\boldsymbol{\sigma}^{\boldsymbol{M}}$, contributions as: $\boldsymbol{\sigma} = \boldsymbol{\sigma}^{\boldsymbol{N}} + \boldsymbol{\sigma}^{\boldsymbol{M}}$. The Newtonian parcel was modeled by the proposition of Navier and Stokes for a Newtonian fluid. For the magnetic parcel, although there has been some discussions [14–17] regarding the proposition of a constitutive equation for $\boldsymbol{\sigma}^{\boldsymbol{M}}$, we opted to use Rosensweig's [9] model. This model recovers the magnetic effect in the equation of motion for all the latest constitutive models when the superparamagnetism hypothesis is considered. Hence, magnetic stresses were modeled by

$$\boldsymbol{\sigma}^{\boldsymbol{M}} = -\left(\frac{\mu_0 H^2}{2}\right)\boldsymbol{I} + \boldsymbol{B}\boldsymbol{H}, \tag{1}$$

where $\boldsymbol{I}$ is the identity second-rank tensor, $\mu_0 = 4\pi \times 10^{-7} H/m$ is the magnetic permeability of the free space, H is the modulus of the applied magnetic vector field $\boldsymbol{H}$ and $\boldsymbol{B}$ represents the magnetic induction at any point of the fluid, given by $\boldsymbol{B} = \mu_0(\boldsymbol{H} + \boldsymbol{M})$. Here $\boldsymbol{M}$ denotes the magnetization field, which is related to a volumetric average of the projections of the dipole moments in the direction of the applied field. For clarification purposes it is worthwile to notice that the term $\boldsymbol{B}\boldsymbol{H}$ represents a second order tensor given by the dyadic product between vectors $\boldsymbol{B}$ and $\boldsymbol{H}$. In this article when no explicit symbology is used between vector quantities it is assumed that we are computing a dyadic product between them. If we take the divergence of $\boldsymbol{\sigma}^{\boldsymbol{M}}$ as defined in Eq. (1) we get $\nabla \cdot \boldsymbol{\sigma}^{\boldsymbol{M}} = \mu_0 \boldsymbol{M} \cdot \nabla \boldsymbol{H}$, which is the classical Kelvin force $\boldsymbol{f}_{\boldsymbol{m}}$ of the Ferrohydrodynamic continuum formalism. If we assume that the particles respond instantaneously (the physical consequences of this hypothesis will be discussed in details in the next section) to the action of the field we get the superparamagnetism hypothesis ($\boldsymbol{M} = \chi \boldsymbol{H}$). Using this assumption, the Kelvin force may be written after some algebraic manipulation as:

$$\boldsymbol{f}_{\boldsymbol{m}} = \mu_0 \chi \nabla\left(\frac{H^2}{2}\right), \tag{2}$$

which for an uniform field $\boldsymbol{H}$ leads to $\boldsymbol{f}_{\boldsymbol{m}} = \boldsymbol{0}$. This leads to a decoupling between magnetic and hydrodydynamic variables. However, the production term $\boldsymbol{\sigma}^{\boldsymbol{M}}\colon \nabla \boldsymbol{v}$ (related to magnetic effects) in the equation of energy could still be non-null for this scenario. If we consider a fully developed pipe flow in the axial direction $\hat{e}_z$, where $\hat{e}_z \perp \hat{e}_r$, being $\hat{e}_r$ the radial direction, we get

$$\boldsymbol{\sigma}^{\boldsymbol{M}} : \nabla \boldsymbol{v} = \frac{\partial v_z}{\partial r} \mu_0 (1 + \chi) H_z H_r. \tag{3}$$

Note that if $\partial v_z / \partial r$, H_z and H_r are non-null, we could have $\boldsymbol{\sigma}^{\boldsymbol{M}}\colon \nabla \boldsymbol{v} \neq 0$ in the equation of energy. In our previous paper [1] our idea was to isolate this possible mechanism in order to get a functional dependency of the convective heat transfer rates inside the tube with respect to magnetic controllable variables. The expression obtained after some algebraic manipulation was

$$Nu_D = \left[\frac{11}{48} + \frac{108}{315}\lambda_m f(\theta)\right]^{-1} \tag{4}$$

with

$$\lambda_m = \frac{2\mu_0 Q (1 + \chi) H_0^2}{\pi D^2 q_s''} \quad \text{and} \quad f(\theta) = \sin\theta \cos\theta, \tag{5}$$

where Q is the volume flow rate, D is the diameter of the tube and H_0 represents the magnitude of the constant applied field. Note that in the limit $\lambda_m \to 0$ we recover $Nu_D = 4.36$, which is the classical solution for a fully developed laminar duct flow subjected to a constant heat flux boundary condition.

3. Additional discussions regarding our last paper

From a physical perspective we must question the source of this mechanism. Since it comes from the term $\boldsymbol{\sigma}\colon \nabla \boldsymbol{v}$, for interpretation purposes we will split the velocity gradient into its symmetric and anti-symmetric contributions:

$$\nabla \boldsymbol{v} = \frac{\nabla \boldsymbol{v} + (\nabla \boldsymbol{v})^T}{2} + \frac{\nabla \boldsymbol{v} - (\nabla \boldsymbol{v})^T}{2}, \tag{6}$$

where the first term on the right hand side is the deformation strain rate tensor $\boldsymbol{D}$ (symmetric, e.g. $\boldsymbol{D} = \boldsymbol{D}^T$) and the second term represents the rotation rate tensor $\boldsymbol{W}$ (anti-symmetric, e.g. $\boldsymbol{W} = -\boldsymbol{W}^T$). The assumption of a superparamagnetic fluid leads to symmetric magnetic stresses. In this sense the particles would be torque-free and since $\boldsymbol{\sigma} = \boldsymbol{\sigma}^T$, our evident conclusion is that $\boldsymbol{\sigma} : \nabla \boldsymbol{v} = \boldsymbol{\sigma} : \boldsymbol{D}$. We could postulate then, that this production mechanism, which seems to appear in the equation of energy, should be associated with the deformation of particle clusters formed in the microstructure of the suspension. Although this mechanism seems to be clear in its interpretation, we need to discuss its origin in deeper details. The superparamagnetic hypothesis assumes that the particles respond instantaneously to the action of an applied field.