International Journal of Approximate Reasoning 103 (2018) 212–226

Contents lists available at ScienceDirect

International Journal of Approximate Reasoning

www.elsevier.com/locate/ijar

A fast analytical approximation type-reduction method for a class of spiked concave type-2 fuzzy sets ☆

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ARTICLE INFO

Article history:
Received 31 October 2017
Received in revised form 9 September 2018
Accepted 4 October 2018
Available online 9 October 2018

Keywords:
Concave type-2 fuzzy sets
Triangular type-2 fuzzy sets
Trapezoidal type-2 fuzzy sets
Type-reduction
Spikes

ABSTRACT

In this paper, an analytical type-reduction method called concave analytical type-reduction method with spikes (CATRS) of two kinds of spiked concave type-2 fuzzy sets is proposed. The concave type-2 fuzzy sets which are considered in the paper include triangular type-2 fuzzy set, trapezoidal type-2 fuzzy set and its spiked versions. The analytical type-reduction method is free of the following steps, such as discourse partition and discourse refinement for algorithms' discrete implementation, thus its calculation complexity is reduced. To obtain the formulae of concave triangular type-2 fuzzy set, the method proposed by Starczewski is extended from the convex case to the concave one, secondary membership function with spikes is considered as a special form of triangular fuzzy set. Centroid set is formed by all the type-1 fuzzy sets that is derived from the convex part of the primary membership with or without spikes at a given discourse partition point. Moreover, the method proposed for concave triangular type-2 fuzzy set is extended to concave trapezoidal type-2 fuzzy set with a trapezoidal fuzzy number approximation operator. The proposed analytical type-reduction methods can address a more general class of type-2 fuzzy sets, it is more convenient for type-2 fuzzy modeling and inference, and the application of type-reduction method to variable universe of discourse controller design shows that the approximation method is applicable for the adaptive fuzzy controller design.

1. Introduction

Triangular type-2 fuzzy sets, whose complexity is between interval type-2 fuzzy sets and general type-2 fuzzy sets, are more suitable for complex fuzzy modeling because of its balance ability between complexity reduction and information preservation. During the past decade, most researches have focused on the properties of type-2 fuzzy sets. To name a few, a manageable and simplified formula for operations on triangular type-2 fuzzy sets within the framework of set-theoretic operations on polygon type-2 fuzzy sets has been studied [1]. Starczewski studied triangular type-2 fuzzy logic systems (FLSs) with approximate extended t-norms operations [2,3]. An approximation operator of real parameters associated to a

☆ This work is supported by the Natural Science Foundation of Heilongjiang Province of China (No. QC2016094) and the National Natural Science Foundation of China (No. 61603126 and No. 11771111).
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https://doi.org/10.1016/j.ijar.2018.10.002
0888-613X/

fuzzy number that preserves the properties of scale and translation invariance, additivity and continuity has been studied [4], and type-2 fuzzy systems can also be used for fuzzy system control design [5,6].

Centroid is a common defuzzification method for fuzzy sets which are defined on the real line, and thus it is practically important in type-1 and type-2 FLSs as a method of defuzzification and type-reduction. To the centroid computation of interval type-2 fuzzy sets, a direct and iteration-free approach based on derivatives has been proposed, and the lower and upper bounds of the centroid can be determined correctly [7]. In [8], the nature of Karnik–Mendel (KM) algorithm in the framework of linear fractional programming is revealed, and a Dinkelbach's algorithm-based computationally efficient implementation method is presented. To the Mamdani interval type-2 FLSs, KM algorithm is enhanced by back propagation algorithms and matrix transformation, the challenging mission of computing derivatives in such systems has been solved [9]. Moreover, convergence problem of the KM algorithm and its extensions are studied [10], a theoretical proof is provided to show the global convergence of KM algorithm and its extensions and improvements. To the type-reduction problem of interval type-2 fuzzy systems, sorting is a frequently used approach in the KM algorithm and its variants, which is known to be computationally expensive, some reformulations are proposed for the center of set's type reducer, and the sorting is eliminated from type-reduction method [11]. To the type-reduction of type-2 fuzzy sets, a closed-form Nie–Tan operator, which is actually an accurate method for defuzzifying interval type-2 fuzzy sets, is proposed by just averaging of the upper and lower bounds of the footprint of uncertainty [12]. Taking accounts of singleton spikes in membership function, computational problems arise when membership functions (MFs) have singleton spikes [13]. To the centroid computation methods for fuzzy numbers, membership function and α-cuts are used to the current centroid computation, the method is simple both in expression and computation [14]. The novel thresh-holding aggregation operators [15] produce such MFs with spikes. Such spikes may occur when modeling concepts defined on a real-valued domain, spikes also appear in performing union operations of fuzzy sets in which some have MFs with discrete support and others have support defined on an interval. Based on a modified definition that was proposed to avoid computational problem when spikes present, the method proposed by Aisbett and Rickard could find a more general class of type-2 fuzzy sets [16]. All of these methods use relatively regular membership functions which are at least piecewise smooth. In practical applications, it is difficult to obtain domain experts' precise descriptions for membership functions. Therefore, irregular-shaped membership functions are rarely used. Moreover, spikes may appear under the circumstance that a piece of suddenly happened unusual data in fuzzy system modeling. Taking 3-D crane for example, spikes are the sudden disturbance on the load mass at the beginning, thus, the spikes must be considered for safety. We can model this type of data as characteristic functions, since the spikes only have values at one point, the time span is usually very small to the time cycle, and the moment is important to robustness of fuzzy system. Such spikes are encountered in aggregating and combining fuzzy sets when the support is discrete, and the support of fuzzy sets is defined on an interval.

The main contributions of this study are summarized as: 1) Concave analytical type-reduction method with spikes for triangular type-2 fuzzy set and trapezoidal type-2 fuzzy set is proposed. The analytical type-reduction method is free of the discourse partition and discourse refinement. 2) Starczewski's method is extended to the concave case, and the spiked secondary membership function is considered as a special form of triangular fuzzy set. 3) The concave analytical type-reduction method proposed for concave triangular type-2 fuzzy set is extended to concave trapezoidal type-2 fuzzy set with a trapezoidal fuzzy number approximation operator.

This paper presents a modified definition of the centroid of fuzzy set that avoids the computational problems and reduces the centroid computation to an MF which combines a series of triangular MFs. Enhanced Karnik–Mendel (EKM) algorithm is used to compute the modified centroid of type-2 fuzzy sets whose MFs have spikes. The details of the proposed analytical type-reduction method for spiked concave type-2 fuzzy sets are organized as follows. Section 2 outlines the centroid of convex, concave triangular type-2 fuzzy sets and concave triangular type-2 fuzzy sets with spikes. Section 3 presents the centroid computing process of concave trapezoidal type-2 fuzzy sets and concave trapezoidal type-2 fuzzy sets with spikes. Section 4 carries out examples for type-reduction method for concave type-2 fuzzy sets with spikes. Conclusions are furnished in Section 5.

2. Analytical formulae for type-reduced set of concave triangular type-2 fuzzy sets

Triangular type-2 fuzzy sets present the balance ability between complexity-reduced and information-protected. Liu's approach [17] suffers from a dense discretization of the domain of α, the complexity of type-reduction of concave type-2 fuzzy sets increases exponentially as the discrete points refined. Finally, in this paper, we introduce an approximation analytical formula for the type-reduced set with two runs of the EKM algorithm [18], the method is suitable for type-reduction of concave type-2 fuzzy sets with spikes. In this section, centroid of convex triangular type-2 fuzzy sets is introduced first, and centroid of concave triangular type-2 fuzzy sets is studied based on the convex case. Furthermore, centroid of concave triangular type-2 fuzzy sets with spikes is illustrated.

2.1. Centroid of convex triangular type-2 fuzzy set

To the problem of type-reduction for convex triangular type-2 fuzzy sets, the partition of the primary domain X in the discrete form is denoted as $\{x_1, x_2, \cdots, x_n\}$, let f_k be the secondary membership function of x_k, and can be denoted by