

# Decomposition and analysis of signals sparse in the dual polynomial Fourier transform

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## ABSTRACT

The acoustic waves transmitted through a dispersive environments can be quite complex for decomposition and localization. A signal which is transmitted through a dispersive channel is usually non-stationary. Even if a simple signal is transmitted, it can change its characteristics (phase and frequency) during the transmission through an underwater acoustic dispersive communication channel. Commonly, several components with different paths are received. In this paper, we present a method for decomposition of multicomponent acoustic signals using the dual polynomial Fourier transform and time-frequency methods. In real-world signals, some disturbances are introduced during the transmission. Common form of disturbances are the sinusoidal signals, making some of the frequency domain signal samples unreliable. Since the signal components can be considered as sparse in the dual polynomial Fourier transform domain, these samples can be omitted and reconstructed using the compressive sensing methods. The acoustic signal decomposition and its reconstruction from a reduced set of frequency domain samples is demonstrated on examples.

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## 1. Introduction

The dispersivity in underwater channels has been a challenging topic in the recent years. Many channels with the phenomena of dispersion have been studied. A dispersive channel in underwater acoustics is a system which produces nonlinear signal transformations [1–5]. That is, it shifts the propagating signal in the phase which will cause shifts in frequency and time in the received signal. Another characterization of dispersive channels is that it produces multicomponent signals due to multipath propagation which can occur for various reasons. The main one is the scattering of acoustic signals on the sea bottom.

The received signal in a dispersive channel is different from the transmitted signal. It is a complex and non-stationary signal. Because of the non-stationary nature of these signals, the time-frequency signal analysis is a suitable tool for analysis. It can help in detection, extraction and localization of transmitted signals. The most common tool for the analysis of non-stationary signals is the time-frequency signal analysis [6–13]. A common problem in prac-

tice is strong harmonic disturbances. After these disturbances are removed, the signal components should be reconstructed.

In the theory of sparse signal reconstruction, a signal is sparse if it has only few non-zero components in comparison to the total length of the signal. If the signal is sparse, it can be reconstructed with less measurements [14–18]. The considered acoustic signal is sparse in the dual polynomial Fourier transform (DPFT) domain, and the noisy measurements (impulses) occur in frequency domain. The impulses in frequency domain will introduce sinusoids in time domain. These disturbances are removed, and the signal components can be reconstructed by compressive sensing methods, such as the matching pursuit algorithm. In this paper, we present a method for decomposition of a signal which was transmitted through a dispersive environment.

The paper is organized as follows. In Section 2, the received signal from a dispersive channel will be modelled and explained. In Section 3 basic theory of compressive sensing is introduced. The polynomial Fourier transform for analysis and localization of acoustic signals will be presented in Section 4. Numerical results and conclusions are given in Sections 6 and 7, respectively.

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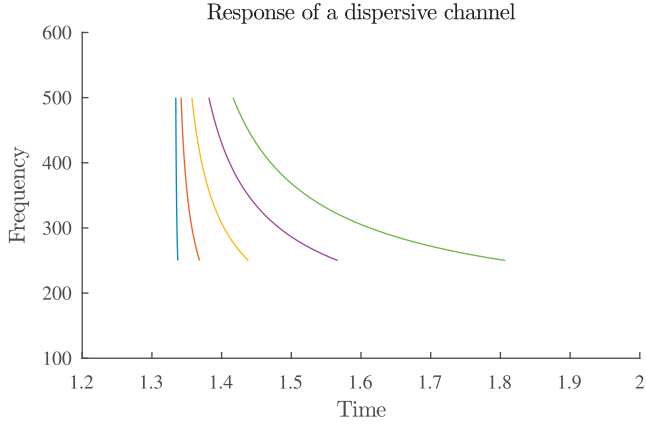


Fig. 1. The time-frequency representation of the impulse response of five modes.

## 2. Modelling of the received signals from dispersive channels

Let us assume that an underwater acoustic wave is transmitted. Assume a linearly frequency modulated (LFM) signal of the form

$$u(n) = e^{j\pi\alpha n^2}. \quad (1)$$

The signal propagates through an isovelocity underwater dispersive channel [2], having the same velocity of sound over all volume [1–5]. We will assume that the transmitter is located at the depth of  $z_t$  meters. The receiver is located at the depth of  $z_r$  meters. The distance between the transmitter and the receiver is denoted by  $r$ . The transfer function of the channel is

$$\begin{aligned} H(f) &= \sum_{m=1}^{+\infty} g_m(z_t) g_m(z_r) \frac{\exp(jk_r(m, f)r)}{\sqrt{k_r(m, f)r}} \\ &= \sum_{m=1}^{+\infty} A_t(m, f, r) \exp(jk_r(m, f)r), \end{aligned} \quad (2)$$

where  $g_m(z_t)$ ,  $g_m(z_r)$  are the modal functions of the  $m$ -th mode for the transmitter and the receiver, respectively. The attenuation rate is  $A_t(m, f, r) = A(m, f)/\sqrt{r}$ . The transfer function depends on the number of modes, and the modes are dependent on wavenumbers  $k_r(m, f)$  [2]

$$k_r(m, f) = \left(\frac{2\pi f}{c}\right)^2 - \left((m - 0.5)\frac{\pi}{D}\right)^2 \quad (3)$$

where  $D$  is the channel depth. The sound speed in the case of underwater communications is  $c = 1500$  m/s. The modal functions  $g_m$  are the solutions [2] of

$$\frac{\partial^2 g}{\partial z^2} + \left(\left(\frac{2\pi f}{c}\right)^2 - k_r^2(m, f)\right)g = 0. \quad (4)$$

It is obvious that the transfer function of a dispersive channel is of a multicomponent structure. The components depend on the wavenumbers  $k_r(m, f)$  and their frequencies, on modal functions  $g_m$  and the distance  $r$ .

The received signal is then

$$x(n) = u(n) * h(n), \quad (5)$$

where  $h(n)$  is the impulse response of (2). An ideal time-frequency representation of the impulse response of a dispersive channel environment is shown in Fig. 1. Our goal is to decompose the mode functions, which will make the problem of detecting the transmitted signal straightforward. This decomposition makes compressive sensing methods application possible to use as well. The decomposition method will be formulated within the compressive sensing approach.

In some real-world scenarios, the signal will be received with a kind of disturbance. Here, we will assume that the signal is corrupted with strong sinusoidal disturbances

$$x_d(n) = x(n) + \sum_{l=1}^{N_M} B_l e^{j(\omega_l n + \psi_l)}. \quad (6)$$

The strong periodic disturbances should be detected and removed. Methods for detecting and removing strong disturbances will be presented next.

## 3. Sparse signal reconstruction

Assume a signal  $x(n)$ ,  $0 \leq n < N$  and its linear transform  $X(k)$ , which will be defined as

$$X(k) = \sum_n \psi_k(n) x(n) \quad (7)$$

where  $\psi_k(n)$  is the basis function of the transform used. In the vector form they are written as

$$\mathbf{x} = [x(0), x(1), \dots, x(N-1)]^T \quad (8)$$

$$\mathbf{X} = [X(0), X(1), \dots, X(N-1)]^T. \quad (9)$$

They are related via  $N \times N$  transformation matrix  $\mathbf{A}_N$  as

$$\mathbf{X} = \mathbf{A}_N \mathbf{x}. \quad (10)$$

We will assume that signal  $x(n)$  is sparse. It means that the signal  $\mathbf{x}$  has only  $K \ll N$  samples  $x(n_1), x(n_2), \dots, x(n_K)$  that are non-zero. When the signal is sparse in one of its domains, it can be reconstructed with less measurements in one of its transformation domains, i.e. with  $N_A < N$ . The signal measurements in this case are coefficients of its transform at positions  $\mathbb{N}_A = \{k_1, k_2, \dots, k_{N_A}\}$ . The measurement vector is defined by

$$\mathbf{y} = [X(k_1), X(k_2), \dots, X(k_{N_A})]^T. \quad (11)$$

Vector form of the measurements equation is

$$\mathbf{y} = \mathbf{A} \mathbf{x} \quad (12)$$

where  $\mathbf{A}$  is a  $N_A \times N$  matrix

$$\mathbf{A} = \begin{bmatrix} \psi_{k_1}(0) & \psi_{k_1}(1) & \dots & \psi_{k_1}(N-1) \\ \psi_{k_2}(0) & \psi_{k_2}(1) & \dots & \psi_{k_2}(N-1) \\ \vdots & \vdots & \ddots & \vdots \\ \psi_{k_{N_A}}(0) & \psi_{k_{N_A}}(1) & \dots & \psi_{k_{N_A}}(N-1) \end{bmatrix} \quad (13)$$

where  $\psi_k(n)$  are the transform coefficients. The matrix is obtained by keeping only the rows of  $\mathbf{A}$  corresponding to the available measurements.

The goal of compressive sensing is to reconstruct the signal by minimizing  $\mathbf{x}$  using the available measurements  $\mathbf{y}$

$$\min \|\mathbf{x}\|_0 \quad \text{subject to} \quad \mathbf{y} = \mathbf{A} \mathbf{x}. \quad (14)$$

It is assumed that the reconstruction conditions are met. The solution of problem (14) can be found in various ways. One of the common algorithms to solve the problem is the orthogonal matching pursuit (OMP) [18]. In the first step of the OMP, the position of the largest component is found

$$n_1 = \arg \max \{\mathbf{x}_0\} \quad (15)$$

using the initial estimate  $\mathbf{x}_0 = \mathbf{A}^H \mathbf{y}$ , calculated using only the available measurements. A new partial matrix of the matrix  $\mathbf{A}$  is formed, omitting all columns except the row which corresponds to the estimated position  $n_1$ . New matrix is then  $\mathbf{A}_1$ . The estimate of the first component in the time domain is

$$\mathbf{x}_1 = (\mathbf{A}_1^H \mathbf{A}_1)^{-1} \mathbf{A}_1^H \mathbf{y}. \quad (16)$$

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