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Transportation Research Procedia 14 (2016) 313 – 322

Transportation Research Procedia

www.elsevier.com/locate/procedia

6th Transport Research Arena April 18-21, 2016

Transit operations with deterministic optimal fare and frequency control

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Abstract

Fare and frequency determination is essential in transit operations. In this paper, passenger demand is analysed and embodied under distributed passengers' willingness to pay. The transit operator's objective is defined as a weighted combination of operator's profit and consumer surplus. The closed-form optimal solutions of transit fare and frequency are obtained in the aim of operator's objective. Different cost structures of the transit system are taken into consideration concerning the marginal effect of passenger demand on transit operating cost. The optimal fare and frequency control strategies are then derived in response to a deterministic changing environment where demand fluctuations are caused by the changes of exogenous factors. For stable or gradually varying situations of exogenous factors, they are determined by implicit equations based on optimal solutions; while for abrupt changing situations, they are determined by the gradient field of the transit operator's objective. A case study for daily optimal operation control of a rapid transit service is demonstrated.

Peer-review under responsibility of Road and Bridge Research Institute (IBDiM)

Keywords: passenger demand; willingness to pay; fare; frequency; operation control

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Peer-review under responsibility of Road and Bridge Research Institute (IBDiM)
doi:10.1016/j.trpro.2016.05.025

1. Introduction

Transit systems, as an important way of human transportation, have drawn much attention both in research and practice over the years. Transit operations are complicated. Different kinds of modes (e.g. airline, bus, ferry and rail) are operated under various environments. A universal prescription that applies in every circumstance may not exist. Nonetheless, there are some common features in the operation processes. An essential concern for the operator is to determine the most suitable fares and frequencies.

There are already a lot of researches concentrated on transit fares and frequencies. Readers can refer to Mohring (1972), Nash (1978), Jansson (1980), Jansson (1993), Jara-Díaz and Gschwender (2003), Pedersen (2003) and so on, to name a few. However, some of the studies are restricted to the supply side; some are confined in a general analysis which is quite abstract. Few of them comprehensively considered the exact and mutual interactions between fare, frequency and passenger demand.

This paper presents another way of thinking. It tries to fill the gap and answer the following questions. First, does there exist closed-form optimal solutions of fare and frequency for transit operations? If there does, what kind of forms can they take? Second, we know that passenger demand will adapt to the changes of fare and frequency accordingly. How to specifically describe the adaptations? Third, we also know, on the other hand, passenger demand actually fluctuates all the time due to the changes of not only endogenous factors but also exogenous factors. How should the transit operator respond to demand fluctuations and adjust its fare and frequency?

For the above interests and purposes, passenger demand and its relationship with fare and frequency are analysed in Section 2 under the assumption of distributed passengers' willingness to pay. The closed-form optimal solutions in the aim of the operator's objective are presented. In Section 3, the optimal fare and frequency control strategies are then derived in response to a deterministic changing environment where passenger demand fluctuations are caused by the changes of exogenous factors. A case study for daily optimal operation control of a rapid transit service is provided in Section 4, while the concluding remarks are given in Section 5. The nomenclatures used in this study are listed in the following text box.

Nomenclature

C	Operating cost of the transit
F	Transit frequency (reciprocal of transit headway)
P	Transit fare per capita
R	Realized passenger demand rate
$\mathbf{X}$	A vector of exogenous demand variables
$c(.)$	A function of passengers' generalized cost of travel
$G(.)$	Cumulative distribution function (CDF) of willingness to pay for the group of transit passengers
$\bar{G}(.)$	Complementary cumulative distribution function or tail distribution of $G(.)$
$g(.)$	Probability density function (PDF) of willingness to pay for the group of transit passengers
$Q(\mathbf{X})$	Potential passenger demand rate determined by exogenous demand variables
z	The objective function of the transit operator
α	A weight parameter indicated in the operator's objective function
β	Passengers' value of time
δ	A fare and frequency independent value composed of other costs of travel
ϕ	Linear parameter associated with operating cost and transit frequency
π	Transit operator's profit
ψ	Consumer surplus for transit passengers