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Frequency-dependent absorption and transmission boundary for the finite-difference time-domain method

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ABSTRACT

In frequency-domain analyses, absorption and transmission characteristics are often modeled as complex surface impedance and complex transfer impedance, respectively. It is however difficult to take the frequency-dependent characteristics into account directly in the finite-difference time-domain (FDTD) method. In this study, a locally reactive boundary using mechanical mass-damper-spring (MDS) systems, which is herein called an MDS boundary, is formulated for sound absorption and transmission analyses by the FDTD method. In addition, the stability conditions of the MDS boundary are discussed. One-dimensional numerical examples show that the MDS boundary can approximate various simple absorption and transmission frequency-dependent characteristics by tuning the parameters of masses, damping constants, and spring constants. Some of them also show that the stability condition of the mechanical MDS system itself is not sufficient and the stability condition considering the effects of the adjacent cells to the MDS boundary can offer the stable calculations. Furthermore, the procedure in a situation where the MDS boundary is not located in parallel with the cell grids is verified by a three-dimensional numerical example.

1. Introduction

The finite-difference time-domain (FDTD) method, which uses staggered grids and a leap-frog algorithm in calculation, is one of the finite-difference methods in which the derivatives in its governing equations are approximated by difference quotients with discretization of time and space. This method was originally developed in the field of electromagnetics by Yee [1], and recently has been applied to various wave propagation problems [2,3]. In acoustic fields, this method can easily provide transient distributions of sound pressure and particle velocity, and therefore the FDTD method has been used to carry out various visualizations of sound fields [4]. In addition, auralizations have been conducted using the obtained transient responses of sound pressure in the evaluation of auditoriums [5], and to investigate physical phenomena such as flutter echoes [6].

In real situations, boundaries often introduce a substantial degree of complexity into systems, as they can bring several different physical mechanisms into play, such as absorption, transmission, resonance, and coupling with vibrations, and therefore, comprehensive models are required in order to accurately predict their behavior. For example, a very thin absorptive layer installed in a large sound field has to be discretized into small cells in order to rigorously calculate its absorption performance: consequently, huge computational resources are required when the attached sound field is discretized into the same small cells. In order to reduce the calculation cost, methods using local-grid systems [7] have been proposed; however, to this point, calculation instability and prediction inaccuracies have been major obstacles hindering development of these methods. In practical terms, a thin layer such as that considered here should be modeled as a boundary without discretization in the thickness direction.

In frequency-domain analyses, an absorptive layer installed in front of rigid walls is often modeled as a boundary with frequency-dependent complex surface impedance [8]. It is, however, difficult to take the frequency-dependent characteristics into account directly in time-domain calculations such as the FDTD method. In addition, further difficulties arise with the typical FDTD method when considering a phase delay due to the absorptive layer: Although the FDTD method is only able to handle real numbers, the phase delay is expressed as an imaginary part of the complex surface impedance. To address these difficulties, a number of boundaries for the finite-difference method have been proposed [9–14]. Among them, Sakamoto et al. [14] have proposed a boundary with two degrees of freedom (DOF) using mechanical mass-damper-spring (MDS) systems, showing that various

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frequency-dependent absorption characteristics can be approximated for the FDTD method by tuning the six parameters of two masses, two damping constants, and two spring constants. However, the previously proposed boundaries cannot deal with sound transmission through them; therefore, it would be desirable to develop a boundary that is able to account for both sound absorption and transmission characteristics of a boundary, allowing for effective application of the FDTD method to sound insulation problems. Extending the idea of Sakamoto's boundary [14], the present paper proposes a locally reactive boundary using MDS systems inserted between propagation media to approximate frequency-dependent absorption and transmission characteristics: the boundary, which is herein called an MDS boundary, has three DOF with seven parameters of three masses, two damping constants, and two spring constants. Note that, although the MDS system has three-DOF, it has only two eigenvalues. This implies that the approximation accuracy is not much different from the Sakamoto's boundary which has the same number of eigenvalue.

Currently, it is well known that stability conditions for wave propagation in FDTD calculations are important. However, instability due to boundary treatment has not been widely investigated. Carpenter et al. [15] investigated the stability characteristics of various high-order compact difference schemes for Euler equations: it was shown that the stability depends on the combination of finite-difference orders for an internal region and boundaries. As a result, they were able to develop a series of compact fourth- and sixth-order schemes that provide stable calculations. Ilan et al. [16] reported an instability of elastic wave propagation due to a free boundary: by considering the impact of the instability of Poisson's ratio of a medium adjacent to the free boundary, a stable approximated formulation for the free boundary was proposed. However, the relationship between the spatial and time intervals was not stated, and thus the stability with an arbitrary spatial interval cannot be assured using this formulation. As is shown in Section 4.1, when an MDS boundary is employed, the calculated responses sometimes diverge, even if the stability condition for wave propagation is satisfied. Thus, it is necessary to derive the associated stability conditions.

In this study, an MDS boundary offering approximate frequency-dependent absorption and transmission characteristics to those which one intends to give the boundary, which are herein called target characteristics, is formulated for the FDTD method. The procedure in a situation where the MDS boundary is not located in parallel with the cell grids is also introduced. In addition, stability conditions for the MDS boundary are derived from the viewpoint of the relationship between spatial and time intervals. Lastly, one- and three-dimensional numerical examples are presented and discussed in terms of the validity of the formulation and stability conditions.

2. Formulation

Let us consider a discretized sound field with an MDS boundary as shown in Fig. 1, where m_1, m_2, and m_3 are the surface densities, x_1, x_2, and x_3 are the displacements of masses, c_1 and c_2 are the

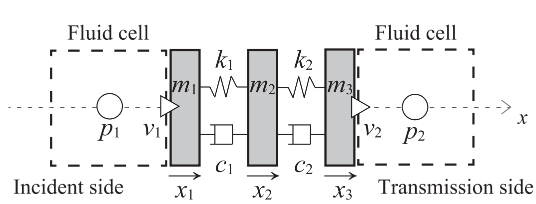

Fig. 1. Discretized sound field with an MDS boundary.

damping constants per unit area, and k_1 and k_2 are the spring constants per unit area, the motion equation for each mass can be written as

$$p_1 = m_1\frac{\partial^2 x_1}{\partial t^2} + c_1\frac{\partial(x_1 - x_2)}{\partial t} + k_1(x_1 - x_2), \tag{1}$$

$$c_1\frac{\partial(x_1 - x_2)}{\partial t} + k_1(x_1 - x_2) = m_2\frac{\partial^2 x_2}{\partial t^2} + c_2\frac{\partial(x_2 - x_3)}{\partial t} + k_2(x_2 - x_3), \tag{2}$$

$$c_2\frac{\partial(x_2 - x_3)}{\partial t} + k_2(x_2 - x_3) = m_3\frac{\partial^2 x_3}{\partial t^2} + p_2, \tag{3}$$

where p_1 and p_2 are the sound pressures defined at the centers of the adjacent cells of incident and transmission sides and t is time. The boundary conditions can be expressed as

$$v_1 = \frac{\partial x_1}{\partial t}, \tag{4}$$

$$v_2 = \frac{\partial x_3}{\partial t}, \tag{5}$$

where v_1 and v_2 are the particle velocities on the boundaries. Eqs. (1)–(3) can be approximated by the central difference scheme as

$$p_1^n = m_1\frac{x_1^{n+1} - 2x_1^n + x_1^{n-1}}{\Delta t^2} + c_1\frac{x_1^{n+1} - x_1^{n-1} - x_2^{n+1} + x_2^{n-1}}{2\Delta t} + k_1\left(x_1^n - x_2^n\right), \tag{6}$$

$$\begin{aligned} &c_1\frac{x_1^{n+1} - x_1^{n-1} - x_2^{n+1} + x_2^{n-1}}{2\Delta t} + k_1\left(x_1^n - x_2^n\right) \\ &= m_2\frac{x_2^{n+1} - 2x_2^n + x_2^{n-1}}{\Delta t^2} + c_2\frac{x_2^{n+1} - x_2^{n-1} - x_3^{n+1} + x_3^{n-1}}{2\Delta t} + k_2\left(x_2^n - x_3^n\right), \end{aligned} \tag{7}$$

$$\begin{aligned} &c_2\frac{x_2^{n+1} - x_2^{n-1} - x_3^{n+1} + x_3^{n-1}}{2\Delta t} + k_2\left(x_2^n - x_3^n\right) \\ &= m_3\frac{x_3^{n+1} - 2x_3^n + x_3^{n-1}}{\Delta t^2} + p_2^n, \end{aligned} \tag{8}$$

where n is the time step and Δt is the time interval of discretization. According to Eqs. (4) and (5), the particle velocities on the boundaries of incident and transmission sides can be approximated by

$$v_1^{n+1/2} = \frac{x_1^{n+1} - x_1^n}{\Delta t}, \tag{9}$$

$$v_2^{n+1/2} = \frac{x_3^{n+1} - x_3^n}{\Delta t}. \tag{10}$$

Displacements x_1, x_2, and x_3 of a new time step can be obtained from Eqs. (6)–(8) and, subsequently, particle velocities on the boundaries can be updated by substituting these displacements into Eqs. (9) and (10). By using the velocities, p_1 and p_2 of a new time step can be obtained with the FDTD calculation.

Additionally, a situation where an MDS boundary is not located in parallel with the cell grids are calculated following the procedure shown in Ref. [14], which is depicted in Fig. 2. First, the boundary is discretized with a staircase approximation, and the particle velocity in the direction normal to the original boundary at a particle-velocity reference point is calculated from the sound

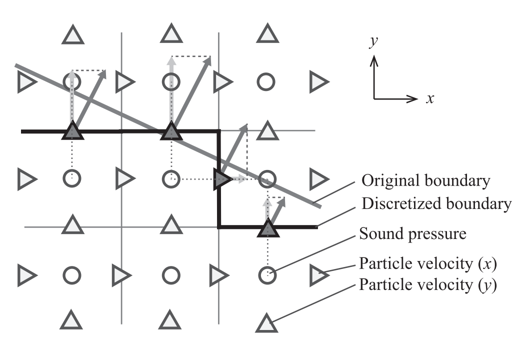

Fig. 2. An MDS boundary located in an orientation non-parallel to the cell grids.