# Profit maximization by a private toll road with cars and trucks

Xiaolei Guo [a,*], Da Xu [b]

[a] *Odette School of Business, Cross-Border Institute, University of Windsor, Ontario, Canada*
[b] *Department of Mechanical, Automotive & Materials Engineering, University of Windsor, Ontario, Canada*

## A R T I C L E   I N F O

## A B S T R A C T

This paper examines the profit maximizing behavior of a private firm which operates a toll road competing against a free alternative in presence of cars and trucks. Trucks differ from cars in value of time (VOT), congestion externality, pavement damage, and link travel time function. We find that the firm takes either a car-strategy or a truck-strategy for profit maximization. For a traffic mix with relatively large car volume and small truck volume, the car-strategy results in no trucks using the toll road, while the truck-strategy results in all trucks using the toll road. We derive the equilibrium flow pattern under any combination of car-toll and truck-toll, based on which we identify a profit-maximizing frontier and a strategy-switching frontier in the car-toll and truck-toll two-dimensional space. By geometrically comparing the two frontiers, we establish general conditions under which each strategy will be taken, which suggest that the truck-to-car VOT ratio, the total traffic demand, and the difference in travel distance between the two roads are critical in shaping the firm's strategy.

© 2016 Elsevier Ltd. All rights reserved.

## 1. Introduction

Privately operated toll roads are increasing around the world. Profit maximization is typically the goal of the private firm which operates the road. For the government, understanding the profit-oriented behavior of the firm is necessary for choosing suitable regulations. There is an extensive literature on modeling the profit-maximizing behavior of private toll roads (see, e.g., Lindsey and Verhoef, 2001; Yang and Huang, 2005; Small and Verhoef, 2007; Tsekeris and Voß, 2009). However, to the best of our knowledge, profit maximization by a private toll road that serves cars and trucks has not been studied. The extension is important for two reasons. First, trucks differ from cars in three major ways: value of time (VOT), congestion externality as measured by passenger car equivalence (PCE), and pavement damage. Previous studies have considered either homogeneous road users or user heterogeneity in VOT only. Second, truck traffic could be an important source of revenue or profit for a toll road operator. For example, the Illinois Tollway implemented a 40% toll increase for trucks in 2015, which was expected to contribute about 60% of its nearly $154 million increase in toll revenue (Chicago Tribune, 2015).

In spite of the importance of trucks in profit generation for (private) toll roads, the literature on toll roads with trucks is mostly focused on system efficiency and policy, not on profit maximization. There is a stream of literature dedicated to truck-only toll lanes and tollways, which studied various aspects of truck toll lanes, including policy and implementation

---

(Samuel et al., 2002), economic and financial feasibility (Holguín-Veras et al., 2003), selection of potential truck-only toll lanes (Chu and Meyer, 2008), and safety benefits (Chu and Meyer, 2010). Most of the research on truck use of toll roads consists of empirical studies and case studies. For example, Zhou et al. (2009) used interviews and survey data to understand truckers' use and non-use of toll roads, Swan and Belzer (2010) studied the truck traffic diversion from the tolled Ohio Turnpike, and a recent report (Geiselbrecht et al., 2015) reviewed many studies on truck use of toll roads in Texas.

A few papers have employed analytical models of toll roads with cars and trucks, all from a socially optimal perspective. Arnott et al. (1992) developed a model with two parallel routes and two user types that could be interpreted as cars and trucks, while they restricted their analyses to two group of car users with different VOTs and/or trip-timing preferences. De Palma et al. (2008) adopted the model to investigate the benefits of separating cars and trucks. They compared the effectiveness of different methods including lane access restrictions, differentiated car and truck tolls, and toll lanes for either cars or trucks. Holguín-Veras and Cetin (2009) used the multinomial logit model to formulate the discrete choice of time of travel for multi-class traffic (cars, small and large trucks) on a single corridor. They computed the socially optimal tolls and discussed the policy implications.

In summary, while a few papers have empirically estimated revenue-maximizing or profit-maximizing truck tolls (e.g., Chu and Meyer, 2008; Swan and Belzer, 2010), none has systematically modeled and analyzed the profit-maximizing behavior of a private toll road with cars and trucks. In this paper, we examine thoroughly the profit-maximizing behavior of a private firm which operates a toll road competing against a free alternative in presence of cars and trucks. Trucks differ from cars in VOT, PCE, pavement damage, and link travel time function. We find that the firm takes either a car-strategy or a truck-strategy for profit maximization. For a traffic mix with relatively large car volume and small truck volume, the car-strategy results in no trucks using the toll road, while the truck-strategy results in all trucks using the toll road. We derive the equilibrium flow pattern under any combination of car-toll and truck-toll, based on which we identify a profit-maximizing frontier and a strategy-switching frontier in the car-toll and truck-toll two-dimensional space. By geometrically comparing the two frontiers, we establish general conditions under which each strategy will be taken, which suggest that the truck-to-car VOT ratio, the total traffic demand, and the difference in travel distance between the two roads are critical in shaping the firm's strategy. The results could be used to explain the pricing strategies of real-world private toll roads such as Highway 407 in Toronto, which has long been criticized that it sets high truck tolls to discourage truck usage (e.g., Canadian Shipper, 2002).

The remainder of the paper is organized as follows. Section 2 presents the general model and derives the equilibrium flow pattern under any combination of car-toll and truck-toll. Section 3 analyzes the profit-maximization problem and establishes general results. Section 4 applies the general results to an instance of the model in which travel distance is explicit, and provides numerical examples. Finally, concluding remarks are given in Section 5.

## 2. The general model

Consider two roads or links, link 1 and link 2, connecting one origin and one destination, where link 1 is a private toll road and link 2 is a free public road. There are two groups of vehicles, cars and trucks, indexed by $g = L, H$, where $L$ and $H$ represent cars (light vehicles) and trucks (heavy vehicles), respectively. Total travel demands of cars and trucks are fixed, and denoted $v_L$ and $v_H$ respectively. Let $v_{gi}$ be the traffic volume of group $g$ on link $i$, $g = L, H$, $i = 1, 2$, where $v_{g1} + v_{g2} = v_g$, $g = L, H$. If $n$ is the congestion PCE of trucks, then the total PCE units on link $i$ is $N_i = v_{Li} + nv_{Hi}$, $i = 1, 2$. Let $t_{gi}(N_i)$ be the travel time of group $g$ on link $i$, which is assumed to be an increasing and continuously differentiable function of $N_i$. Note that the fixed travel costs independent of travel time (e.g., distance-based vehicle operating and fuel costs) of each link can be converted into a fixed component of the travel time function. The variable travel costs depending on travel speed (e.g., the variable component of fuel costs depending on speed) can also be incorporated into the travel time function, because travel speed can be written as a function of travel time.

Let $P_L$ and $P_H$ be the tolls charged on cars and trucks, respectively, and $\beta_L$ and $\beta_H$ be their values of time (VOT). The equilibrium conditions are

$$t_{g1}(N_1) + \frac{P_g}{\beta_g} \leq t_{g2}(N_2), \text{ if } v_{g1} > 0, \quad g = L, H \tag{1}$$

$$t_{g1}(N_1) + \frac{P_g}{\beta_g} \geq t_{g2}(N_2), \text{ if } v_{g2} > 0, \quad g = L, H \tag{2}$$

$$v_{g1} + v_{g2} = v_g, \quad g = L, H \tag{3}$$

$$v_{Li} + nv_{Hi} = N_i, \quad i = 1, 2 \tag{4}$$

$$v_{gi} \geq 0, \quad g = L, H, \quad i = 1, 2 \tag{5}$$

Condition (1) stipulates that, if link 1 is used by group $g$ in equilibrium, then the generalized travel time (including toll) of link 1 for group $g$ must not exceed the travel time of link 2. Condition (2) is interpreted similarly.