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Convexity and robustness of dynamic traffic assignment and freeway network control

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ABSTRACT

We study the use of the System Optimum (SO) Dynamic Traffic Assignment (DTA) problem to design optimal traffic flow controls for freeway networks as modeled by the Cell Transmission Model, using variable speed limit, ramp metering, and routing. We consider two optimal control problems: the DTA problem, where turning ratios are part of the control inputs, and the Freeway Network Control (FNC), where turning ratios are instead assigned exogenous parameters. It is known that relaxation of the supply and demand constraints in the cell-based formulations of the DTA problem results in a linear program. However, solutions to the relaxed problem can be infeasible with respect to traffic dynamics. Previous work has shown that such solutions can be made feasible by proper choice of ramp metering and variable speed limit control for specific traffic networks. We extend this procedure to arbitrary networks and provide insight into the structure and robustness of the proposed optimal controllers. For a network consisting only of ordinary, merge, and diverge junctions, where the cells have linear demand functions and affine supply functions with identical slopes, and the cost is the total traffic volume, we show, using the Pontryagin maximum principle, that variable speed limits are not needed in order to achieve optimality in the FNC problem, and ramp metering is sufficient. We also prove bounds on perturbation of the controlled system trajectory in terms of perturbations in initial traffic volume and exogenous inflows. These bounds, which leverage monotonicity properties of the controlled trajectory, are shown to be in close agreement with numerical simulation results.

1. Introduction

The System Optimum (SO) Dynamic Traffic Assignment (DTA) problem, introduced in Merchant and Nemhauser (1978a, 1978b), has attracted significant interest from the transportation research community, see, e.g., (Peeta and Ziliaskopoulos, 2001) for an overview. While originally proposed mainly for planning purposes, it is also being increasingly used as a framework to compute optimal control for traffic flow over freeway networks, e.g., see Gomes and Horowitz (2006); Muralidharan and Horowitz (2012), when traffic controllers aim to minimize a global cost of the whole network – hence Social Optimality, as opposed to the single-vehicle oriented User Equilibrium modeling frameworks. Continuing along these

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relatively recent trends, this paper focuses on the use of solutions of two variants of the SO-DTA to design optimal controls for dynamic network traffic flows over a given time horizon, in the form of variable speed limit, ramp metering, and routing (turning ratios) matrices.

The Cell Transmission Model (CTM), originally proposed in Daganzo (1994, 1995), is a compelling framework to simulate realistic first order traffic dynamics. It consists of a time and space discretization of the kinematic wave models of Lighthill-Whitham and Richards (Lighthill and Whitham, 1955; Richards, 1956). Unfortunately, straightforward formulations of DTA for the CTM are known to lead to non-convex problems, and hence are unsuitable especially for real-time applications. On the other hand, in the DTA formulation of Ziliaskopoulos (2000), the supply and demand constraints of the CTM are relaxed to yield a linear program. However, the computational simplicity resulting from this relaxation comes at the expense of possible infeasibility of a resulting optimal solution with respect to traffic dynamics. Quite interestingly, (Muralidharan and Horowitz, 2012) shows that the optimal solution of a linear program analogous to the DTA relaxation in Ziliaskopoulos (2000) can be realized exactly for traffic dynamics modeled by the link-node cell transmission model by proper design of ramp metering and variable speed limit controller, when demand functions are linear, supply functions are affine, and the network consists of a mainline with on- and off-ramps. In this paper, we consider extensions of the approach proposed in Gomes and Horowitz (2006); Muralidharan and Horowitz (2012) to arbitrary networks, where traffic dynamics is inherited by the CTM, with traffic dynamics encompassing the ones originally proposed in Daganzo (1994, 1995), and also allowing for arbitrary concave demand and supply functions, and convex cost functions, including total travel time, total travel distance, and total delay as special cases. Under these generalizations, the resulting SO-DTA is a convex program, which can be solved using readily available software tools such as `cvx` (Grant and Boyd, 2008; 2014) and possibly suited for distributed iterative solvers, e.g., see our preliminary work (Ba et al., 2015).

In short, the considered approach to the optimal control of freeway networks over a finite time horizon consists of two steps: (i) to formulate and solve convex optimal control problems that are relaxations either of the DTA problem (with turning ratios part of the optimization), or of the Freeway Network Control (FNC) problem (where the turning ratios are exogenously imposed); (ii) to design open-loop variable speed limits, ramp metering, and routing controls over the time horizon to make such optimal solution feasible with respect to traffic dynamics modeled by the CTM. Natural questions concern: (a) under which conditions step (ii) above is not necessary, i.e., the optimal solution of the convex optimal control problem is readily feasible with respect to traffic flow dynamics; and (b) how robust the optimal control computed through the procedure above is with respect to perturbations to the network during the execution of the open-loop controller over the time horizon. The main novel contributions of this paper address questions (a) and (b) as follows. On the one hand, using Pontryagin's maximum principle, we prove that, for networks consisting only of ordinary, merge, and diverge junctions, and whose cells have linear demand and affine supply functions with identical slopes, the optimal solution of the FNC problem with total traffic volume as cost does not require the use of variable speed limits as a proper choice of ramp metering controls makes it readily feasible with respect to the CTM model of Daganzo (1994). On the other hand, we provide bounds on the perturbation to the system trajectory under the open-loop controller obtained from the two-step procedure due to perturbations in the initial traffic volumes and exogenous inflows. In order to derive such bounds, we use the fact that the nominal (i.e., unperturbed) controlled system trajectory resulting from steps (i) and (ii) above is always in free-flow, and hence it satisfies a certain monotonicity property that in turn can be used to evaluate its robustness to perturbations of the initial traffic volumes and exogenous inflows.

It is helpful to clarify the novelty of our contributions with respect to existing literature. We generalize the applicability of the two-step procedure of using solution of the two DTA variants for design of optimal traffic flow control to general network topologies, concave supply and demand functions, convex cost functions, than the ones considered previously, e.g., in Gomes and Horowitz (2006); Muralidharan and Horowitz (2012).[1] The maximum principle has been used to identify necessary conditions for optimal control of traffic flow over networks, e.g., in Friesz et al. (1989). However, the underlying model for traffic flow dynamics in Friesz et al. (1989) does not capture backward propagation of congestion, and in particular does not resemble CTM.

Robustness of open-loop controllers can be quantified through standard sensitivity analysis of controlled traffic dynamics. Our bounds, which exploit the monotonicity properties of controlled system trajectories, are applicable to relatively larger perturbations than those obtained through such standard techniques. Implications of such monotonicity property for robustness of dynamic network flows have been recently investigated in different contexts (Como et al., 2015; 2013a; 2013b). Our robustness analysis of the solution to deterministic DTA problems is to be contrasted with chance-constrained solution of stochastic SO-DTA, e.g., in Waller and Ziliaskopoulos (2006) under probabilistic information about inflows. Our approach to applying maximum principle and robustness bounds necessitates consideration of continuous time versions of CTM in the analytical part of the paper. Such continuous time and discrete space versions have been used previously, e.g., to develop probabilistic versions of CTM (Jabari and Liu, 2012). Our adoption of continuous time version is merely to facilitate analysis, and is not to be interpreted as a new numerical framework for traffic flow dynamics. Indeed, the simulations reported in this paper are performed in the standard discrete time discrete space version of CTM.

[1] In particular, considering general convex demand and supply curves (as opposed to piecewise affine ones) allows one to deal with models where the average velocity is a continuous function of the traffic volume (as opposed to discontinuous piecewise constant): these include, e.g., the Greenshields affine velocity model (Greenshields, 1935) with corresponding quadratic fundamental diagram.