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Facility location design under continuous traffic equilibrium

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ABSTRACT

This paper presents two modeling approaches for median-type facility location design under elastic customer demand and traffic equilibrium in a continuous space. The first approach, following the continuum approximation scheme, builds upon the special case of an infinite homogeneous plane where traffic equilibrium can be described by an ordinary differential equation. The solution to this homogeneous case, sometimes in a closed form, is then used to develop approximate solutions to more general cases (e.g., those in a heterogeneous space). This model provides a computationally efficient way to obtain managerial insights and near-optimal solutions, especially for large problem instances. We also develop a more traditional discrete location model in the form of a mixed-integer program, which builds directly upon a nonlinear partial differential equation description of customer traffic equilibrium. We develop a Lagrangian relaxation based solution approach with an embedded finite-element method subroutine, to separate and solve the location decisions as well as the traffic equilibrium. Numerical experiments are conducted to illustrate applicability of the proposed models and to compare performance of the two complementing modeling approaches.

1. Introduction

The median-type facility location problem seeks optimal facility locations to minimize the transportation costs for serving spatially distributed customers, subject to a budget constraint for facility investments. This and related topics have been studied extensively; see Daskin (1995) and Drezner (1995) for comprehensive surveys. In most traditional models, customer allocation to built facilities can often be determined individually based on the smallest transportation cost (distance or time).

In reality, however, customers often choose their own service facility, and their overall costs may be coupled to one another due to shared traffic congestion as well as demand-price equilibria at the facilities. In such cases, the decisions of these customers (i.e., destination and routing) should be modeled endogenously with the decisions on facility locations. A large amount of traffic will be added to the roadway network especially near the neighborhood of the built facilities. The construction of new facilities, therefore, directly induces or diverts day-to-day traffic demand and alters the congestion pattern (and hence transportation costs) in the network. Meanwhile, the decisions on facility locations directly depend on the spatial distribution of customers, the associated transportation costs, and as a result, the likelihood of attracting customer demand. The congestion caused by customer traffic may result in higher transportation cost and community resistance, which in turn will likely compromise efficiency of these service facilities. Hence, disregarding the decisions of the customers under congestion (especially in areas with heavy background traffic) while selecting facility locations may not only cause

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unnecessarily high transportation cost and low facility patronage, but also impose a negative socioeconomic impact on the general public.

The problem described above can be somewhat related to a large body of literature on facility location problems with queuing delay. Most of these studies incorporate queuing models to capture the expected service time (Larson, 1974, 1975; Berman et al., 1985; Zhang et al., 2009, 2010). Earlier models aim at achieving a certain level of service by either providing redundant coverage (Daskin, 1983; Revelle and Hogan, 1989; Ball and Lin, 1993) or explicitly minimizing queuing cost (Marianov and ReVelle, 1996). Later studies integrate the impact of in-facility congestion in location–allocation problems to determine the optimal number and location of facilities, their service capacity, and the assignment of customers (Marianov and Serra, 2002; Aboolian et al., 2008, 2012; Castillo et al., 2009). Minimizing queuing effects has also been considered in competitive facility location problems (Marianov et al., 2008; Zhang and Rushton, 2008). These models generally have a similar assumption that transportation cost can be estimated by simple shortest travel distances and hence no traffic congestion exists.

The impact of traffic congestion on customer access cost and facility location design is gaining more attention especially in recent years. Bai et al. (2011, Accepted for publication) and Hajibabai and Ouyang (2013) examined traffic congestion impact and incorporated shipment routing decisions endogenously in supply chain design problems. Hajibabai et al. (2014) further extended the idea and incorporated the societal impacts of freight traffic on deterioration of highway pavement infrastructures. Traffic routing under congestion was also considered in shelter location problems (Sherali et al., 1991; Li et al., 2012; An et al., 2015). Konur and Geunes (2011, 2012) analyzed a two-stage game to characterize the qualitative effects of traffic congestion costs on supply chain activities in a competitive environment.

These studies mainly are based on mixed-integer nonlinear programming models, which explicitly rely on nonlinear functions to address congestion effects and integer variables to address discrete location and network related decisions. Linearization of congestion cost functions is a standard practice, but it often leads to additional integer variables and further exacerbates computational burden. Hence, such models can only be applied to small or moderate-size problem instances. This motivated alternative modeling approaches such as those in the continuous setting. For example, Yang et al. (1994) and Yang (1996) first formulated continuous user equilibrium model to understand traveler route choices in a continuous space when travel cost is a linear function of the traffic flow density. These models generalize the earlier work by Beckmann and Puu (1985) on equilibrium price and shipment flow distribution by explicitly addressing local congestion. The dual of the equilibrium conditions are derived and solved by finite element method techniques. Yang and Wong (2000) and Wong and Yang (1999) further extended this school of work by allowing more general nonlinear transportation cost functions, elastic customer demand, and demand-dependent service fees at the facilities. The customers' destination and travel path choices at equilibrium are described by nonlinear partial differential equations. Later a series of extensions considered variants to the problem, including those explicitly addressing customers' choices on facilities (Wong and Sun, 2001), trip origin locations (Ho and Wong, 2005), and multiple customer classes (Ho et al., 2003). These innovative models provide an alternative way to describe traffic equilibrium, but yet these existing work all assumes that the locations of facilities are given in advance (although subject to competition for customer patronage).

Coincidentally, in the facility location literature, a school of continuum approximation (CA) models (Newell, 1971, 1973; Daganzo, 1984a,b; Daganzo and Newell, 1986; Ouyang and Daganzo, 2006) have also been developed to provide good approximate solutions to large-scale logistics problems. These models have been applied to a variety of location and routing problems to avoid the excessive computational burden from the traditional discrete models. See Langevin et al. (1996) and Daganzo (1991) for earlier reviews, and Li and Ouyang (2010), Cui et al. (2010), Wang and Ouyang (2013) for more recent applications. The development of the CA models and the emergence of continuous traffic equilibrium models have provided a unique opportunity to build holistic continuum models that integrate both facility location decisions and traffic equilibrium in a continuous space.

This paper first develops a continuous facility location design framework that explicitly addresses induced customer traffic and congestion under spatial-price equilibrium. The model builds upon the special case of an infinite homogeneous plane, where the optimal solution of customer traffic equilibrium can be reduced from a partial differential equation to a much simpler ordinary differential equation, which, for some simple cases, can be solved (approximately) in closed-forms. Then, this solution is used as a building block to develop approximate solutions to more general heterogeneous cases. In contrast to most CA models which study uncapacitated fixed-charge location problems, this paper chooses to focus on developing a solution approach for median-type problems. For comparison, we also propose a discrete formulation that directly builds upon the partial differential equation description of traffic equilibrium, and we develop a Lagrangian relaxation (LR) based solution approach to decompose the problem into a location subproblem and a traffic equilibrium subproblem. Numerical experiments are conducted to illustrate applicability of both the CA model and the discrete model and to draw insights through comparisons. Traffic congestion is found to have a significant impact on the system cost and optimal facility location design.

It shall be noted that the model formulation and solution methodology developed in this paper are generic. They can be used to address problems that arise in other types of settings, as long as the problem involves facility location decisions, traffic congestion, and equilibrium. This is especially the case when customer traffic can be described by ordinary or partial differential equations in the continuum. Examples include those involving pedestrian traffic in open space (Xia et al., 2008) or airspace congestion in the aviation systems (Loo et al., 2005). The corresponding location problems may be on the