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A study of interval analysis for cold-standby system reliability optimization under parameter uncertainty

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ABSTRACT

This paper presents a study of interval analysis for solving cold-standby system reliability optimization problems with considering parameter uncertainty. Most works reported in existing literature have been based on the assumption that the probabilistic properties and statistical parameters have a known functional form, which is usually not the case. Very often the parameters are presented in form of an interval-valued number or bounds/tolerance from the engineering design. In this paper, interval analysis is used to incorporate this in the system optimization problems. A definition of interval order relation reflecting decision makers' preference is proposed for comparing interval numbers. A computational algorithm is developed to evaluate the system reliability and expected mission cost, in which a discrete approximation approach and a technique of interval universal generating function are used. For illustration, an application to sequencing optimization for heterogeneous cold-standby system is given; a modified genetic algorithm is developed to solve the proposed optimization problem with interval-valued objective. The results indicate that the interval analysis exhibits a good performance for dealing with parameter uncertainty of cold-standby system optimization problems.

1. Introduction

The development of industrial technology involves an increasing amount of design of complex and interrelated systems. Reliability is an important performance measure of industrial systems, especially when it is of safety–critical concerns. Extensive research has been carried out on system reliability optimization; survey papers (Kuo & Prasad, 2000; Kuo & Wan, 2007; Tillman, Hwang, & Kuo, 1977) have summarized many earlier studies on reliability optimal problems.

In the existing literature of system reliability optimization, most results are based on assumptions that the probabilistic properties or parameters of time-to-failure are deterministic. However, due to observation difficulties, resource limits and system complexity, uncertainties are usually unavoidable while modeling real industrial systems. For many engineering problems, it is overly difficult or costly to collect sufficient data about the uncertainties, especially at the very beginning of design processes. Stakeholders and decision makers have to deal with a variety of uncertainty issues when making decisions without sufficient information. In fact, many parameters are specified as intervals of some kind in engineering design.

Bayesian approach (Howson & Urbach, 2006) could be used to study the uncertainties associated with the estimation of parameters of a probability distribution (Pasanisi, Keller, & Parent, 2012; Srivastava & Deb, 2013; Troffaes, Walter, & Kelly, 2014). The unknown parameters are assumed to be random variables. With the Bayesian approach, subjective judgments are required to estimate the Bayesian random variables. The estimation of the Bayesian random variables can be improved when more data become available. Before receiving more data, however, the Bayesian approach remains a subjective representation of uncertainty. Fuzzy theory is another commonly used method for analysing uncertainty issues (Dotoli, Epicoco, Falagario, & Sciancalepore, 2015; Hanss & Turrin, 2010; Wang & Watada, 2009). In the fuzzy approach, the imprecise parameters are represented as fuzzy numbers. However, the fuzzy sets and their membership functions are required to be known. It is a formidable task for decision makers to specify the appropriate membership functions in advance.

In order to overcome the drawbacks of probabilistic methods and fuzzy approaches, interval analysis first developed by Moore (1966) has recently received some attention. The interval analysis has been used to deal with problems of uncertainty in diverse

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Nomenclature

Notations

$[a]$	interval-valued number
$\underline{a}$	the lower bound of $[a]$
$\bar{a}$	the upper bound of $[a]$
ρ_0	level of decision maker's preference
ρ	ratio of two interval numbers in the inclusion type relation
$R_j(t), R(t)$	reliability of component j and system, respectively
$E_C(t)$	expected cost of system
N	number of components in the system
T_j	random variable representing the time-to-failure of component j
$F_j(\cdot)$	cumulative distribution function of T_j
$f_j(\cdot)$	probability density function of T_j
V_j	start-up cost of component j
w_j	running cost of component j per time unit
Y_j	cumulative working time of first j components
$s(j)$	index of component j in the predetermined order
τ	mission time of system
Δ	duration of each time interval
m	number of mission time intervals
$[p_j(i)]$	probability that component j fails in the time interval $[\Delta i, \Delta(i+1)]$
$t_{j,i}$	i-th realization of T_j
$u_j(z)$	u-function representing discrete distribution of T_j
$U_j(z)$	u-function representing discrete distribution of Y_j

Table 1
Component parameters for the optimization example.

Component	$[\eta_{1,\cdot}]$	$[\eta_{2,\cdot}]$	V	w
1	[57, 62]	[1.00, 1.05]	100	3.0
2	[78, 82]	[1.70, 1.90]	80	3.5
3	[65, 75]	[1.20, 1.40]	210	1.2
4	[34, 36]	[1.00, 1.10]	150	3.0
5	[130, 145]	[2.30, 2.50]	220	2.0
6	[77, 82]	[1.75, 1.85]	60	3.7
7	[78, 83]	[1.15, 1.25]	120	1.8
8	[46, 54]	[1.05, 1.20]	70	2.5
9	[72, 77]	[1.15, 1.25]	100	1.5
10	[69, 71]	[1.35, 1.65]	180	2.0

fields, such as circuit analysis (Kolev, 1993), damage identification (Wang, Yang, Wang, & Qiu, 2012), structure safety analysis (Impollonia & Muscolino, 2011; Wang, Gao, Song, & Zhang, 2014; Zhang, Dai, Beer, & Wang, 2013), electric power system (Pereira & Da Costa, 2014), and so on. In these studies, interval variables were used to quantitatively describe the uncertain parameters in the face of limited information. Up to now, research on interval uncertainty problems has concentrated mainly in the aforementioned fields, while the application of interval analysis to system reliability optimization for complex industrial systems is relatively new.

In the existing literature of system reliability optimization, Feizollahi and Modarres (2012) suggested a robust deviation framework to deal with uncertain component reliabilities in constrained redundancy allocation problems; they addressed uncertainty by assuming that the component reliabilities belong to interval uncertainty sets. However, the interval numbers are not incorporated directly. Gupta, Bhunia, and Roy (2009) and Bhunia, Sahoo, and Roy (2010) dealt with optimization problems for series systems; the reliability of each component was represented as an interval number. Sahoo, Bhunia, and Kapur (2012) studied the constrained multi-objective reliability optimization problem of systems with interval-valued component reliabilities. However, these studies dealt specifically with the series or series–parallel systems with active redundancy and given interval-valued component reliability, or placed greater attention on optimization algorithms.

In this paper, we present a study of interval analysis for cold-standby system optimization problems considering uncertain probabilistic parameters. Our study focuses on the evaluation and optimization of system reliabilities and expected mission costs. A discrete approximation approach based on Levitin et al. (2013) and the interval universal generating function (IUGF) technique (Li, Chen, Yi, & Tao, 2011) are used in the evaluation procedures for estimating the system reliability and the expected mission cost. IUGF is a technique which extends the universal generating function (UGF) (Levitin, 2005) for the situations with interval-valued parameters.

In solving the optimization problem with interval-valued objective, a set of interval values appear during the selection of the best alternative, which leads to a question related to the comparison of two arbitrary interval numbers. In this paper, we define a new order relation for two arbitrary interval numbers considering different levels of decision maker's preference. The level of decision maker's preference is measured by the ratio ρ_0, where $\rho_0 = 1$ stands for neutrality; $\rho_0 > 1$ stands for optimistic preference; otherwise pessimistic.

For purposes of illustration, we propose the application of interval analysis theory to the sequencing optimization problem for heterogeneous cold-standby systems (Levitin et al., 2013a). In this

Table 2
Best obtained components sequences for $\rho_0 = 1$[a] and different τ.

Mission time τ	Optimal initiation sequence	Expected mission cost $[E_C]$	System reliability $[R]$
400	**9, 7, 5, 8, 6, 2, 1, 10, 3, 4**	**[1551.5, 1641.6]**	[0.9677, 0.9839]
	1, 2, 3, 4, 5, 6, 7, 8, 9, 10[b]	[1859.3, 1924.2]	[0.9682, 0.9838]
500	**9, 7, 3, 5, 8, 6, 2, 1, 10, 4**	**[1963.8, 2075.5]**	[0.8451, 0.9035]
	1, 2, 3, 4, 5, 6, 7, 8, 9, 10[b]	[2220.1, 2275.1]	[0.8440, 0.9045]
600	**3, 9, 7, 5, 8, 10, 1, 6, 2, 4**	**[2342.0, 2424.1]**	[0.5955, 0.7269]
	1, 2, 3, 4, 5, 6, 7, 8, 9, 10[b]	[2517.7, 2540.4]	[0.5955, 0.7262]

[a] When $\rho_0 = 1$, the proposed order relation are the same with the definition in Bhunia and Samanta (2014).
[b] Comparative trials with initiation sequence of (1, 2, 3, 4, 5, 6, 7, 8, 9, 10).

Table 3
Best obtained components sequences for $\tau = 500$ and different ρ_0.

ρ_0	Optimal initiation sequence	$[E_C]$	$[R]$
$\rho_0 \leqslant 0.3$	9, 7, 5, 3, 8, 6, 2, 1, 10, 4	[1965.062, 2075.128]	[0.8451, 0.9035]
$0.31 \leqslant \rho_0 \leqslant 2.76$	9, 7, 3, 5, 8, 6, 2, 1, 10, 4	[1963.850, 2075.494]	
$\rho_0 \geqslant 2.77$	3, 9, 7, 5, 8, 6, 2, 1, 10, 4	[1963.831, 2075.545]	