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A comment on "The quadratic programming problem with fuzzy relation inequality constraints"

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ABSTRACT

This note provides counterexamples to illustrate that Definition 1 and Theorem 2 which are proposed by Molai (2012) are incorrect. We first point out some deficiencies and errors found in the above referenced paper. The incorrect of Theorem 2 is due to the inaccuracy of Definition 1. Then we present a counterexample to illustrate the incorrect of FRI path. At last, we propose a new definition of FRI path of max-product fuzzy relation inequality. Some examples are presented to illustrate that the new definition of FRI path is correct.

1. Introduction

Molai (2012) proposed a fuzzy relation quadratic programming model with a quadratic objective function subject to the max-product fuzzy relation inequality constraints as follows:

$$\begin{aligned} \min \quad & z(x) = cx + \frac{1}{2}x^T Qx \\ \text{s.t.} \quad & A \bullet x \geqslant d^1, \\ & B \bullet x \leqslant d^2, \\ & x \in [0,1]^n \end{aligned} \tag{1}$$

where $x = [x_1, \ldots, x_n]^T$ is the vector of decision variables to be determined. The operator of $\bullet$ denotes the max-product composition operator. Let M, N, and L be the index sets of $\{1, \ldots, m\}, \{1, \ldots, n\}$, and $\{1, \ldots, l\}$, respectively. The constraint part of model (1) is to find a set of solution vectors $x \in [0,1]^n$ such that

$$\begin{aligned} & \max_{j \in N}(a_{ij} \cdot x_j) \geqslant d_i^1, \quad i \in M, \\ & \max_{j \in N}(b_{ij} \cdot x_j) \leqslant d_i^2, \quad i \in L. \end{aligned} \tag{2}$$

Denote the set of solution of (2) by $X(A, d^1, B, d^2)$. A solution $\hat{x} \in X(A, d^1, B, d^2)$ is called a *maximum solution* if $x \leqslant \hat{x}$ for all $x \in X(A, d^1, B, d^2)$. Also, a solution $\check{x} \in X(A, d^1, B, d^2)$ is called a *minimal solution* if $x \leqslant \check{x}$, for any $x \in X(A, d^1, B, d^2)$, implies $x = \check{x}$. If the solution set of (2) is not empty, then the maximum solution can be computed by the following relation:

$$\hat{x}_j = \bigwedge_{i=1}^{l} \left\{ \frac{d_i^2}{b_{ij}} | b_{ij} > d_i^2 \right\}, \quad j \in N,$$

where $\bigwedge_{i=1}^{l} \varnothing = 1$ is defined. Define a series of index sets by $J_i = \{j \in N | a_{ij} \cdot \hat{x}_j \geqslant d_i^1\}, i \in M$. Let $\Lambda = J_1 \times J_2 \times \cdots \times J_m$. A vector $p \in \Lambda$ is called a general path or G-path (Guo, Pang, Meng, & Xia, 2013) of (2). Denote the set of all the G-paths of (2) by GP.

Definition 1. A vector $p = (p_1, \ldots, p_m)$ is called an FRI path of the feasible domain of problem (1) if for any $i \in M$,

$$\begin{aligned} & p_i \in J_i, \quad J_i \cap \{p_1, \ldots, p_{i-1}\} = \varnothing, \\ & p_i = 0, \quad \text{otherwise}. \end{aligned}$$

Denote P the set of all the FRI paths of the feasible domain of problem (1). Then Molai proposed similar theorem to Guo and Xia (2006) as follows.

Theorem 1 Molai, 2012. *Suppose the feasible solution set of problem (1) is not empty. Let $p \in P$ be an FRI path of the feasible domain of problem (1). Define $x^p = [x_1^p, \ldots, x_n^p]^T$ by*

$$x_j^p = \bigvee_{i=1}^{m} \left\{ \frac{d_i^1}{a_{ij}} | p_i = j \right\}, \quad j \in N,$$

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where $\bigvee_{i=1}^{m}\varnothing = 0$ is defined. Then the solution set of FRI of the feasible domain of problem (1) is: $X(A, d^1, B, d^2) := \bigcup_{p\in P}\{x | x^p \leqslant x \leqslant \hat{x}\}$.

Proof. The proof is similar to the proof of Theorem 2.3 in Guo and Xia (2006). □

However, there are some deficiencies and errors in Molai (2012) which are presented below:

1. Definition 1 and Theorem 2 in Molai (2012) are incorrect, as we demonstrate in Section 2. The incorrect of Theorem 2 is due to the inaccuracy of Definition 1. The expression of "$J_i \cap \{p_1, \ldots, p_{i-1}\} = \varnothing, p_i = 0$, otherwise." is extra in Definition 1 and it should be removed.
2. The sufficient conditions $c_t > 0, q_{tj} > 0, q_{jt} > 0, \forall j \in N$ are added to the assumptions of Corollary 2. The sufficient conditions $c_t > 0, q_{ts} > 0, q_{st} > 0, \forall t, s \in N$ are added to the assumptions of Corollary 3.
3. Step 9 of Example 3 (p. 261) is error. In Step 9, the author think that the following conditions are satisfied (1) $|J_1^{(1)}| = 1$ and $J_1^{(1)} = \{7\}$, (2) $\frac{d_1^1}{a_{17}} = \frac{0.3}{0.99} \geqslant \frac{d_i^1}{a_{i7}}, \forall i \in I_7^{(1)}$, (3) $c_1^1 = 1.5005 > 0$, $q_{i7}^1 > 0$, and $q_{7i}^1 > 0, \forall i \in \{1,3,4,5,6,7\}$. Therefore, according to Step 9 of algorithm in p. 260, the author set $x_7^* = \frac{0.3}{0.99} = 0.303$. Firstly, it is not Step 9 of algorithm, but Step 8 of algorithm in p. 260. Secondly, the author applies Theorem 3 in Molai (2012) to Step 8. The condition (3) of Theorem 3 or Step 8 is $c_i^1 > 0, q_{ij}^1 > 0$, and $q_{ji}^1 > 0, \forall j \in N$. However, the condition (3) of Step 9 of Example 3 is $c_1^1 = 1.5005 > 0, q_{i7}^1 > 0$, and $q_{7i}^1 > 0, \forall i \in \{1,3,4,5,6,7\}$. It is obvious that the condition (3) of Step 9 of Example 3 is completely different with the condition (3) of Theorem 3 or Step 8. So, we cannot set $x_7^* = \frac{0.3}{0.99} = 0.303$ in this step. Thus, Step 9 of Example 3 is incorrect and hence it should be removed.

2. Examples

In this section, we first present some examples to show that Theorem 2 and the definition of FRI path in Molai (2012) are incorrect. In addition, one example is proposed to illustrate why Theorem 2.3 and the definition of FRI path in Guo and Xia (2006) are correct.

Example 1. We consider the following max-product fuzzy relation inequality

$$\begin{aligned}&\begin{bmatrix}0.66 & 0.89 & 1 & 0.56\\ 0.7 & 0.25 & 0.9 & 0.4\\ 0.75 & 0.88 & 0.6 & 0.8\\ 0.9 & 0.8 & 0.4 & 0.68\end{bmatrix} \bullet \begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix} \geqslant \begin{bmatrix}0.45\\0.4\\0.25\\0.2\end{bmatrix},\\ &\begin{bmatrix}0.25 & 0.4 & 0.1 & 0.4\\ 0.3 & 0.6 & 0.24 & 0.8\\ 0.71 & 0.5 & 0.23 & 0.3\\ 0.6 & 0.8 & 0.5 & 0.6\end{bmatrix} \bullet \begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix} \leqslant \begin{bmatrix}0.141\\0.2\\0.435\\0.333\end{bmatrix},\end{aligned} \tag{3}$$

where • is the max-product composition. The maximum solution of (3) is as $\hat{x} = [0.555, 0.333, 0.666, 0.25]^T$. The sets of J_i's are as: $J_1 = J_2 = \{3\}, J_3 = J_4 = \{1,2,3\}$. $\Lambda = \prod_{i=1}^{4} J_i = \{3\} \times \{3\} \times \{1,2,3\} \times \{1,2,3\}$. By using Definition 1, there is only one FRI path $p = [3,0,0,0]$. The corresponding solution is $x^p = [0,0,0.45,0]$. Obviously, x^p is not a solution of (3). It illustrate that Definition 1 and Theorem 2 in Molai (2012) are incorrect. Now, we compute all of the minimal solutions of (3) by G-paths. All the G-paths are

$p^1 = [3,3,1,1]^T, \quad p^2 = [3,3,1,2]^T, \quad p^3 = [3,3,1,3]^T,$

$p^4 = [3,3,2,1]^T, \quad p^5 = [3,3,2,2]^T, \quad p^6 = [3,3,2,3]^T,$

$p^7 = [3,3,3,1]^T, \quad p^8 = [3,3,3,2]^T, \quad p^9 = [3,3,3,3]^T.$

The corresponding solutions are

$x^1 = [0.333, 0, 0.45, 0]^T, \quad x^2 = [0.333, 0.25, 0.45, 0]^T,$
$x^3 = [0.333, 0, 0.5, 0]^T,$

$x^4 = [0.222, 0.284, 0.45, 0]^T, \quad x^5 = [0, 0.284, 0.45, 0]^T,$
$x^6 = [0, 0.284, 0.5, 0]^T,$

$x^7 = [0.222, 0, 0.45, 0]^T, \quad x^8 = [0, 0.25, 0.45, 0]^T,$
$x^9 = [0, 0, 0.5, 0]^T.$

It implies that the set of minimal solutions of (3) is $\check{X} = \{x^7, x^8, x^9\}$.

Example 2. We consider the following max–min fuzzy relation inequality

$$\begin{aligned}&\begin{bmatrix}0.5 & 0.8 & 0.9 & 0.3 & 0.85 & 0.4\\ 0.2 & 0.2 & 0.1 & 0.95 & 0.1 & 0.8\\ 0.8 & 0.4 & 0.1 & 0.8 & 0.1 & 0.1\\ 0 & 0 & 0.1 & 0 & 0 & 0\end{bmatrix} \bullet \begin{bmatrix}x_1\\x_2\\x_3\\x_4\\x_5\\x_6\end{bmatrix} \geqslant \begin{bmatrix}0.85\\0.6\\0.5\\0.1\end{bmatrix},\\ &\begin{bmatrix}0.4 & 0.7 & 0.95 & 0.4 & 0.9 & 0.5\\ 0.3 & 0.3 & 0.2 & 1.0 & 0.2 & 0.85\\ 0.8 & 0.75 & 0.3 & 0.2 & 0.2 & 0.2\\ 0.2 & 0 & 0 & 0.2 & 0 & 0\end{bmatrix} \bullet \begin{bmatrix}x_1\\x_2\\x_3\\x_4\\x_5\\x_6\end{bmatrix} \leqslant \begin{bmatrix}0.9\\0.8\\0.7\\0.2\end{bmatrix},\end{aligned} \tag{4}$$

where • is the max–min composition. The maximum solution of (4) is $\hat{x} = [0.7, 0.85, 0.9, 0.7, 1.0, 0.8]^T$. For any $i = 1,2,3,4$, compute the J_i by $J_i = \{j \in N | a_{ij} \wedge \hat{x}_j \geqslant d_i^1\}$, one has $J_1 = \{3,5\}, J_2 = \{4,6\}$, $J_3 = \{1,4\}, J_4 = \{3\}$. So, $\Lambda = \prod_{i=1}^{4} J_i = \{3,5\} \times \{4,6\} \times \{1,4\} \times \{3\}$. It follows that all the G-paths are

$p^1 = [3,4,1,3]^T, \quad p^2 = [3,4,4,3]^T, \quad p^3 = [3,6,1,3]^T,$
$p^4 = [3,6,4,3]^T,$

$p^5 = [5,4,1,3]^T, \quad p^6 = [5,4,4,3]^T, \quad p^7 = [5,6,1,3]^T,$
$p^8 = [5,6,4,3]^T.$

By $x_j^p = \bigvee_{i=1}^{m}\left\{d_i^1 | p_i = j\right\}, j = 1,2,\ldots,n, \bigvee_{i=1}^{m}\varnothing = 0$, compute the corresponding solutions:

$x^1 = [0.5, 0, 0.85, 0.6, 0, 0]^T, \quad x^2 = [0, 0, 0.85, 0.6, 0, 0]^T,$
$x^3 = [0.5, 0, 0.85, 0, 0, 0.6]^T,$

$x^4 = [0, 0, 0.85, 0.5, 0, 0.6]^T, \quad x^5 = [0.5, 0, 0.1, 0.6, 0.85, 0]^T,$
$x^6 = [0, 0, 0.1, 0.6, 0.85, 0]^T,$

$x^7 = [0.5, 0, 0.1, 0, 0.85, 0.6]^T, \quad x^8 = [0, 0, 0.1, 0.5, 0.85, 0.6]^T.$

It follows that the set of minimal solutions of (4) is $\check{X} = \{x^2, x^3, x^4, x^6, x^7, x^8\}$. All the FRI paths are