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School timetabling problem under disturbances

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ABSTRACT

School timetables are one or multiple times per year generated to assign class-teacher combinations to class rooms and timeslots. Post-publication disturbances such as absence of teachers typically pose a need for schedulers to rapidly implement some minor changes to avoid empty periods in the timetable. In this paper our aim is to define methods to efficiently solve the school timetabling problem under disturbances. We present three types of solution methods, namely a simple rule-of-thumb, a heuristic and an optimization approach. Exhaustive numerical experiments have been performed with data from five high schools in The Netherlands, each with their unique characteristics in number of classes, number of teachers and number of daily meetings. For each of the three methods we show advantages and disadvantages as well as the effects of resulting changes in the schedules.

1. Introduction

Main decisions in school timetable problems are to make class-teacher-room assignments and allocate meetings to empty slots in a schedule. Schedulers need to take a variety of constraints into account, such as teachers' availability and preferences, room capacity, lesson spread for classes and load balancing for teachers. Observations in practice learn that the generation of timetables is a time consuming process which is executed by software in combination with the manual interaction of a scheduler. The general school timetabling problem is proven to be NP-complete (Even, Itai, & Shamir, 1976). Post et al. (2012) concluded that the field of educational timetabling is nowhere near solving all possible instances of high-school timetabling. Initially, mathematical programming approaches were used in deriving feasible timetables (e.g., Papoutsis, Valouxis, & Housos, 2003; Tripathy, 1984). Later mainly heuristics have been designed (e.g., Fonseca & Santos, 2014; Zhang, Liu, M'Hallah, & Leung, 2010). Typically those methods are intended to be used to design new school timetables from scratch for a (part of) a year. However, timetable users must be able to make minor changes rapidly and easily after publication due to disturbances such as teachers' illness or extracurricular activities. In practice, this rescheduling process is mainly arranged manually. There is a need for new methods to efficiently reschedule parts of school timetables that can be applied at different types of schools (Pillay, 2014). Our aim is to present a model and a variety of solution approaches to define and solve the school timetabling problem under disturbances. Exhaustive experiments are performed to show the outcomes of the different methods, and the ability to generalize outcomes. To this end we use data of five high schools in The Netherlands each with different characteristics instead of as commonly seen in literature only of a single high school (Pillay, 2014).

In the school timetabling problem under disturbances, typically an initial timetable is rescheduled and the altered timetable is compared to the initial timetable. Commonly, in The Netherlands, meetings between absent teachers and their classes will be removed from the schedule and will not be rescheduled in another time period. Consequently, the number of empty periods for classes in the timetable of that day increase. Empty periods are perceived negatively and the overall aim is to keep the number of empty periods as low as possible. Schedulers focus on reducing those resulting additional periods by making short term changes in the schedule. The reduction of empty periods is obtained by temporarily shifting meetings of other teachers to other time periods in the new timetable. However, the reduction of the number of empty periods comes at a cost. Shifting meetings force classes and teachers to adapt to sudden changes of the schedule, which can be experienced as something negative. Therefore, the scheduler has to create a balance between reducing the number of empty periods, keeping the schedule stable, i.e., not deviating too much from the old timetable and being alert on the amount of shifts on a specific day and over days. The latter kind of shifts is typically less valued than shifts on a specific day. Consequently, the quality of the new timetable is determined by its compactness expressed by the number of empty periods, the stability of the

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schedule and the type of shifts. Typically in literature the quality of the schedule is expressed in soft constraints (e.g., Pillay, 2014). In our newly designed rescheduling policies we explicitly incorporate those performance measures in the objective.

The problem of school timetabling under disturbances can be classified in the field of school timetabling. The school timetabling problem, also referred to as the class-teacher model, consists of assigning meetings to periods for a specific class-teacher combination such that no teacher or class is involved in more than one meeting at a time (e.g., Carter & Laporte, 1998; De Werra, 1985). We can roughly divide literature on school timetabling into two categories, namely (1) class-teacher assignment (Asratian & De Werra, 2002; Azmi Al-Betar & Khader, 2012; De Werra, Asratian, & Durand, 2002); and (2) class scheduling to assign meetings for a specific subject for each class to timeslots and rooms (Burke, Mareček, Parkes, & Rudová, 2012; Sampson, Freeland, & Weiss, 1995; Sampson & Weiss, 1995). Some papers (Al-Yakoob & Sherali, 2007; Alvarez-Valdes, Martin, & Tamarit, 1996) address those decision problems in a sequential way. To our knowledge no methods are specifically designed to perform limited alterations to already published school timetables.

The structure of this paper is as follows. We define the school timetabling problem under disturbances in Section 2. Section 3 presents the different solution approaches, namely a simple rule-of-thumb, an optimal approach and a heuristic. Section 4 shows for a specific setting the outcomes of the different methods. Experiments and data collection at 5 different schools are defined in Section 5. Results and numerical insights are shown in Section 6. Finally, we present conclusions in Section 7.

2. Problem definition

In this section, we formally define and formulate the school timetabling problem under disturbances. Two types of school timetabling problems can be considered, namely one where all pupils in a class follow exactly the same meetings and one where pupils in a class may attend different meetings (Post et al., 2012). In the lower classes in the educational system in The Netherlands all pupils in a class follow exactly the same meetings, which is also the case for some other countries (e.g., the 11- to 14- year-olds in English Secondary Schools). For an overview of the timetabling problem in different countries we refer to Post et al. (2012). In this paper, we focus on the lower classes of high schools where all pupils in a class follow exactly the same meetings. Specifically for those pupils it is the general understanding that the amount of empty periods should be as low as possible.

A specific set of disturbances can be represented by changes in teachers' availability. A published timetable is available showing an assignment of teachers to classes and time periods. This assignment will be input in the school timetabling problem. If a teacher is unavailable, no other teacher will take over to teach this subject to a class. Given that the related meeting is not scheduled at the start or end of a day, we define the resulting time slot as an empty period. If the canceled meeting was scheduled at the start of the end of the day, the students will start/end their day later/earlier. In a feasible schedule sufficient room capacity is available to match meetings to rooms. Given that, we do not consider subject-room assignment decisions in the school timetabling problem under disturbances. Consequently, the model aims to re-allocate *meetings* for each teacher-class combination to a timeslot given the new information on teachers' availability. The goal is, as explained in Section 1, to minimize a weighted sum of the number of empty periods and the number of shifts made between the old and the new schedule. As mentioned in the introduction, we make a distinction between the number of shifts on a specific day and over days.

In defining the parameters and variables we follow where applicable the notation as presented by Santos, Uchoa, Ochi, and Maculan, 2010. The following set of parameters is defined:

C: Set of classes;
T: Set of teachers;
D: Set of days;
P: Set of time periods on a day, where for each day the time periods are numbered from 1 to $|P|$;
$\tilde{R}$: Requirement matrix, where $\tilde{r}_{tc}$ specifies the number of meetings involving teacher t and class c, excluding the disturbed meetings;
$\tilde{T}$: Availability matrix, where $\tilde{t}_{tdp} = 1$ if teacher t is available at time period p of day d, $\tilde{t}_{tdp} = 0$ otherwise;

$$\bar{x}_{tcdp} = \begin{cases} 1 & \text{if teacher } t \text{ and class } c \text{ meet at time period } p \text{ of day } d \text{ in the old schedule and teacher } t \text{ is not disturbed at time period } p \text{ of day } d \\ 0 & \text{otherwise;} \end{cases}$$

w_1: Penalty for each empty period;
w_2: Penalty for the shift of a meeting to another time period;
w_3: Penalty for the shift of a meeting to another day;

The decision variables and auxiliary variables are defined as follows:

$$x_{tcdp} = \begin{cases} 1 & \text{if teacher } t \text{ and class } c \text{ meet at time period } p \text{ of day } d, \\ 0 & \text{otherwise;} \end{cases}$$

$h_{cd} \in \mathbb{Z}_+$: Number of empty periods for class c at day d;
$\overline{a}_{cd} \in \mathbb{Z}_+$: Time period of the first meeting of class c at day d;
$\underline{a}_{cd} \in \mathbb{Z}_+$: Time period of the last meeting of class c at day d;
$g_{tcd} \in \mathbb{Z}_+$: Number of meetings between teacher t and class c shifted to day $d$from another day;
s_{tcdp}: Binary variable equal to one if a meeting between teacher t and class c is shifted to time period p at day d.

3. Solution approaches

In this section we define three different solution approaches to solve the timetabling problem under disturbances. First, we define a simple rule-of-thumb that can be performed manually without the need of a computer. Secondly, we construct an integer linear programming model (ILP) that solves the problem to optimality. Given the complexity of the problem, we finally define a heuristic procedure that can generate results efficiently. In Section 6 we will compare the different methods to analyze the changes in the schedules obtained.

3.1. Simple rule-of-thumb

A simple rule-of-thumb to solve the timetabling problem under disturbances can be described as follows: pick the first of the empty periods caused by a disturbance and try to shift the last or first meeting of the day to the empty period. Whenever this is not possible, try to find another time period at this day whose scheduled meeting can be shifted to the empty period and where the last or first meeting of the day can be shifted to. Whenever this is not possible either, check whether a meeting at the end or start of another day can be shifted to the empty period. In Appendix A, the pseudocode for this rule-of-thumb is given. The description of the parameters and variables not described in the pseudocode, can be found in Section 2.