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Prediction-based relaxation solution approach for the fixed charge network flow problem

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ABSTRACT

A new heuristic procedure for the fixed charge network flow problem is proposed. The new method leverages a probabilistic model to create an informed reformulation and relaxation of the FCNF problem. The technique relies on probability estimates that an edge in a graph should be included in an optimal flow solution. These probability estimates, derived from a statistical learning technique, are used to reformulate the problem as a linear program which can be solved efficiently. This method can be used as an independent heuristic for the fixed charge network flow problem or as a primal heuristic. In rigorous testing, the solution quality of the new technique is evaluated and compared to results obtained from a commercial solver software. Testing demonstrates that the novel prediction-based relaxation outperforms linear programming relaxation in solution quality and that as a primal heuristic the method significantly improves the solutions found for large problem instances within a given time limit.

1. Introduction

The fixed charge network flow problem (FCNF) can be described on a network $G = (N, A)$, where N and A are the sets of nodes and edges, respectively. Each node $i \in N$ has a supply/demand commodity requirement R_i ($R_i > 0$ if node i is a supply node; $R_i < 0$ if node i is a demand node; otherwise, $R_i = 0$). Each edge $(i,j) \in A$ has costs associated with commodity flow. Let c_{ij} and f_{ij} denote the variable and fixed costs, respectively. An artificial capacity value, M_{ij}, can be used in the problem formulation to ensure that the fixed cost f_{ij} is incurred whenever there is a positive flow on $(i,j) \in A$. There are two types of variables in the FCNF, edge (i,j) flow and usage, denoted as x_{ij} and y_{ij}, respectively. The latter is a binary variable representing the decision to use $(i,j) \in A$ for routing commodities and incurs the fixed cost f_{ij}. The formulation is provided in Eqs. (1)–(4).

$$\min \sum_{(i,j)\in A} (c_{ij}x_{ij} + f_{ij}y_{ij}) \tag{1}$$

$$\text{s.t.} \sum_{(i,j)\in A} x_{ij} - \sum_{(j,i)\in A} x_{ji} = R_i \quad \forall i \in N \tag{2}$$

$$0 \leqslant x_{ij} \leqslant M_{ij}y_{ij} \qquad \forall (i,j) \in A \tag{3}$$

$$y_{ij} \in \{0,1 \qquad \forall (i,j) \in A \tag{4}$$

The objective function in (1) is the sum of variable and fixed costs incurred for the solution. Constraint (2) ensures that the inflow and outflow satisfy the supply or demand requirements at node $i \in N$. Constraint (3) creates a logical relationship between x_{ij} and y_{ij}. Constraint (4) defines y_{ij} as binary, which makes the problem a mixed binary programming problem.

Many practical problems can be modeled as a FCNF problem or variation thereof, such as transportation problems (El-Sherbiny & Alhamali, 2013), the lot sizing problem (Steinberg & Napier, 1980), the facility location problem (Melo, Nickel, & Saldanha-da Gama, 2009; Nozick, 2001), network design (Costa, 2005; Ghamlouche, Crainic, & Gendreau, 2003; Lederer & Nambimadom, 1998) and others (Armacost, Barnhart, & Ware, 2002; Jarvis, Rardin, Unger, Moore, & Schimpeler, 1978). The FCNF problem is an NP-hard problem and over the decades, a significant quantity of research has been directed towards providing solution approaches to the FCNF. Many techniques utilize variations of the branch-and-bound (B&B) algorithm to search for the exact solution of the FCNF (Barr, Glover, & Klingman, 1981; Cabot & Erenguc, 1984; Driebeek, 1966; Hewitt, Nemhauser, & Savelsbergh, 2010; Kennington & Unger, 1976; Ortega & Wolsey, 2003; Palekar, Karwan, & Zionts, 1990). The B&B algorithm may be inefficient due to the lack of the tight bounds during the linear programming (LP) relaxation.

State-of-the-art MIP solvers combine a variety of cutting plane strategies, heuristic techniques with B&B to search for the exact optimal solution. Modern MIP solvers use preprocessing methods to reduce the search space and significantly accelerate the solving

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processes (Bixby, Fenelon, Gu, Rothberg, & Wunderling, 2000). More details of the preprocessing techniques can be found in Nemhauser and Wolsey (1988), Wolsey (1998), Fügenschuh and Martin (2005), Mahajan (2010). Additionally, due to the impractical computational effort to obtain exact solutions for large instances, heuristic approaches to obtain near-optimal solutions have generated considerable research interest (Adlakha & Kowalski, 2010; Antony Arokia Durai Raj & Rajendran, 2012; Balinski, 1961; Kim & Pardalos, 1999; Molla-Alizadeh-Zavardehi, Hajiaghaei-Keshteli, & Tavakkoli-Moghaddam, 2011; Monteiro, Fontes, & Fontes, 2011; Sun, Aronson, McKeown, & Drinka, 1998).

In this study, a novel *prediction-based relaxation* (PBR) method is proposed. PBR may be used as an FCNF heuristic solution technique or as a primal heuristic to potentially improve on exact search solution quality and time. The new method leverages a probabilistic model to create an informed reformulation and relaxation of the FCNF problem. For the FCNF problem, a binary probabilistic classification model is required. Approaches to develop such models are common in the field of statistical learning and include logistic regression, linear discriminant analysis, partial least squares discriminant analysis, and random forests among others. The present work demonstrates the utility of PBR by implementing one such model, a logistic regression developed by Nicholson and Zhang (2016).

The primary contribution of this work is to introduce and evaluate a new paradigm for approaching the classical FCNF problem. In particular, elements from statistical learning are combined with more traditional solution techniques from Operations Research. Section 2 provides a brief review of the predictive model from Nicholson and Zhang (2016). Subsequently, the necessary mathematical transformations to leverage the predictive model results are described and the formal PBR mathematical model is presented. Section 3 describes the experiments to evaluate PBR as an independent heuristic and explores the strategy to combine PBR and the B&B algorithm as an exact solution approach. The work is concluded in Section 4.

2. Prediction-based relaxation

2.1. Motivation

The prediction-based relaxation approach is dependent on an underlying statistical model to produce probabilities for positive edge flow in an FCNF instance. A variety of predictive modeling methods to produce such probabilities are possible. One such possibility is the statistical learning model developed in Nicholson and Zhang (2016). Their model is based on the 28 variables described in Table 1. The authors classify these variables as being associated with four types of network characteristics: overall network level (e.g., total number of nodes), edge specific attributes (e.g., variable cost), linear relaxation based variables (e.g., edge flow in an relaxed problem), and lastly, variables related to the nodes incident to an edge (e.g., node degree). While many network and network component features are possible, their guiding principle was to find predictive variables that are relatively straightforward and easy to compute since their goal was to introduce and explore a new paradigm for understanding and exploiting FCNF problems. Through the use of initial trial and error tests and secondly, the use of Akaike information criterion informed stepwise regression, this small subset was selected. These features are used as predictors in a logistic regression model to predict components of the FCNF optimal solution. The associated regression coefficients are provided in Table 1. In particular, the model produces a likelihood of edge usage with a logistic regression model. The response variable is a binary variable, indicating whether or not the edge has positive flow in the optimal solution. Let Y_{ij} denote the response variable,

Table 1
Logistic regression model from Nicholson and Zhang (2016).

Predictor description	Predictor notation	Regression coefficient
	(Intercept)	8.32
Number of nodes	n	−0.048
Number of edges	m	−0.0055
Variable cost on (i,j)	c_{ij}	−0.0879
Fixed cost on (i,j)	f_{ij}	−0.0002
Fixed to variable cost ratio	γ_{ij}	<0.0001
Tail node type: transhipment	$t_i = 0$	−0.543
Tail node type: supply	$t_i = 1$	2.17
Head node type: transhipment	$t_j = 0$	−3.01
Head node type: supply	$t_j = 1$	−2.38
Outdegree of tail node	$\bar{d}_{i*}$	−2.59
Indegree of tail node	$\bar{d}_{*i}$	−2.69
Outdegree of head node	$\bar{d}_{j*}$	−2.3
LP relaxation usage	$\bar{l}^{B}_{ij}$	1.4
LP relaxation flow	$\bar{l}_{ij}$	5.76
Tail node requirements	$\bar{r}_i$	0.967
Tail adjacent in-supply	$\bar{r}^{S}_{i*}$	0.765
Tail adjacent in-demand	$\bar{r}^{D}_{i*}$	0.878
Tail adjacent out-supply	$\bar{r}^{S}_{*i}$	−0.215
Tail adjacent out-demand	$\bar{r}^{D}_{*i}$	0.464
Head adjacent in-supply	$\bar{r}^{S}_{j*}$	−0.797
Head adjacent in-demand	$\bar{r}^{D}_{j*}$	−0.917
Head adjacent out-supply	$\bar{r}^{S}_{*j}$	−0.122
Demand head nodes adjacent to tail	$\bar{d}^{D}_{i*}$	−1.3
Supply tail nodes adjacent to tail	$\bar{d}^{S}_{*i}$	1.84
Demand tail nodes adjacent to tail	$\bar{d}^{D}_{*i}$	4.57
Demand head nodes adjacent to head	$\bar{d}^{D}_{j*}$	−1.58
Supply tail nodes adjacent to head	$\bar{d}^{S}_{*j}$	3.19
Demand tail nodes adjacent to head	$\bar{d}^{D}_{*j}$	1.26

$$Y_{ij} = \begin{cases} 1, & \text{edge } (i,j) \text{ is used in the FCNF optimal solution} \\ 0, & \text{otherwise.} \end{cases}$$

Logistic regression is supervised learning technique to develop a probabilistic classifier. The resulting model, based on the input data, assigns a probability that the response variable attains a given value. PBR requires a probabilistic classifier, albeit it is in no way limited to logistic regression derived models. Ideally, a probabilistic model for PBR is both highly accurate and efficient to implement. The logistic regression model conceived and studied in Nicholson and Zhang (2016) meets both of these criteria. Ideally, if edges used in an optimal FCNF solution could be predicted perfectly, then the B&B search would be altogether eliminated. Perfect predictions however are not a reasonable aspiration in general. If, however, a predictive model were highly accurate, then at least the search space could potentially be reduced or the search otherwise informed. The goal of PBR is to exploit such probability models.

2.2. Mathematical model

In this work, the probability from the final logistic regression model is used to develop a novel heuristic for the FCNF. Let $p_{ij} \approx P(Y_{ij} = 1)$ denote the probability estimate that edge $(i,j) \in A$ is used in the FCNF problem. The value for p_{ij} is derived from any appropriate predictive model. Let $c'_{ij} = -\ln p_{ij}$ where $0 < p_{ij} \leqslant 1 \; \forall (i,j) \in A$. The PBR cost function is denoted by z'_{PBR} and defined as

$$z'_{\text{PBR}} = \sum_{(i,j)\in A} c'_{ij} x_{ij} = \sum_{(i,j)\in A} -x_{ij} \ln p_{ij} = -\ln \prod_{(i,j)\in A} p_{ij}^{x_{ij}}$$