Computers & Industrial Engineering 99 (2016) 223–228

Contents lists available at ScienceDirect

Computers & Industrial Engineering

journal homepage: www.elsevier.com/locate/caie

# Machine repairing systems with standby switching failure

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ARTICLE INFO

*Article history:*
Received 19 March 2015
Received in revised form 18 April 2016
Accepted 16 July 2016
Available online 19 July 2016

*Keywords:*
Optimization
Probabilistic global search Lausanne method
Sensitivity analysis
Supplementary variable technique

ABSTRACT

This paper attempts to explore the performance measures and optimization analysis of machine repairing systems with standby switching failure. The time between failures and time-to-repair of failed machines are assumed to be exponential and general distributions, respectively. The standby is switched over to the operating mode when an operating machine fails, and the switch is subject to failure with probability *q*. A recursive method based on the supplementary variable technique is used to derive the steady-state probabilities of failed machines in the system. We then develop various performance measures of the system and construct the expected cost function per unit time. The probabilistic global search Lausanne method is employed to determine the optimal number of standbys and the optimal repair rate, wherein the expected cost per unit time is minimized. Finally, some numerical examples are provided for illustrative purposes.



## 1. Introduction

The machine repairing system can be applied to a variety of real situations, such as computer network, telecommunications, coal shipment, aircraft maintenance, and many others (see Haque & Armstrong, 2007). Gupta and Srinivasa Rao (1994, 1996) used the supplementary variable technique to calculate the stationary probability distribution for the M/G/1 machine repairing system with no spares and cold standbys. Using the same technique, Srinivasa Rao and Gupta (2000) obtained the probability distribution of the number of down machines in the M/G/1 machine interference system with cold-, warm- and hot-standbys. Wang and Kuo (1997) presented the profit analysis of the $M/E_K/1$ machine repair problem with non-reliable service station. Wang, Ke, and Ke (2007) considered the M/M/R machine repair problem with balking, reneging and standby switching failure. Ke and Lin (2010) employed the maximum entropy method to compute the stationary probability distribution of the number of down machines approximately for the M/G/1 machine repair problem. A multi-server machine repair problem with warm standbys under synchronous multiple vacation policy was investigated by Ke and Wu (2012). Wang, Liou, and Lin (2013) studied the issue of the M/M/R machine repair problem with imperfect coverage and service pressure condition. Liou, Wang, and Liou (2013) considered the controllable M/M/2 machine repair problem under the triadic (0, Q, N, M) policy. Wang, Liou, and Wang (2014) investigated a multiple-vacation M/M/1 standby machine repair problem with an unreliable repairman. Recently, Wang, Su, and Yang (2014) utilized a recursive method based on the supplementary variable technique to develop steady-state analytical solutions in the M/G/1 machine repair problem with multiple imperfect coverage.

In the past, many studies on machine repairing systems with standbys assumed that it is perfect to switch over the standby to operating one. However, in real-life situations, the possibility of failure to switch a standby to an operating one exists. In this paper, we utilize the supplementary variable technique to analyze an M/G/1 machine repairing system with switching failure. Huang, Lin, and Ke (2006) examined a parametric nonlinear programming approach for a repairable system with switching failure and fuzzy parameters. Wang, Dong, and Ke (2006) performed a comparative analysis of the reliability and availability of four systems with warm standby units, reboot delay and standby switching failures. Ke, Lee, and Ke (2008) studied the reliability and sensitivity of a system with standby switching failure. Wang and Chen (2009) compared the availability of three different systems with reboot delay and standby switching failures. Liu, Ke, Hsu, and Hsu (2011) applied the bootstrap method to investigate the statistical inferences of a repairable system with standby imperfect switching. Hsu, Ke, and Liu (2011) studied the statistical inferences of an availability system with reboot delays, standby switching failures and unreliable service station. An M/M/R machine repair

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http://dx.doi.org/10.1016/j.cie.2016.07.016
0360-8352/

problem with a switching failure probability, reboot delay and repair pressure coefficient has been examined and can be seen in the recent work of Hsu, Ke, Liu, and Wu (2014).

Existing works on the machine repairing system with switching failure mainly focus on the specific repair time distribution (e.g., exponential distribution). However, from a practical viewpoint, the repair time can be an arbitrary random variable. This motivates us to investigate a machine repairing system with switching failure and distribution-free repair times. The contributions of this research are outlined as follows: (1) we examine the machine repairing system with standby failure, in which the repair time follows general distribution; (2) we propose a solution procedure to compute the steady-state probability distribution of the number of failed machines in the system; and (3) the cost optimization problem is formulated to find the optimal number of standbys, and optimal repair rate to minimize the expected cost per unit time.

## 2. Model description and notations

We consider a machine repairing system with $N = M + W$ homogeneous machines, where $M$ machines are operating and $W$ machines are spare machines (standbys). When an operating machine breaks down, it is replaced by an available standby. Suppose that the switch is subject to failure with probability $q$ during the switching from the standby state to the operating state. The time-to-failure of the operating machine is assumed to be exponentially distributed with parameter $\lambda$. Whenever one of the operating machines fails, it is immediately replaced by a standby, as long as one is available. The time between failures of standby machines obeys an exponential distribution with parameter $\alpha$. Whenever an operating machine or a standby fails, it is immediately sent to a repair and is repaired in order of breakdown. The repair times of the failed machines are independent and identically distributed (i.i.d.) random variables, having a general distribution $B(v)(v \geqslant 0)$, a probability density function $b(v)(v \geqslant 0)$ and mean repair time $b$. The repairman can repair only one failed machine at a time. When a failed machine is repaired and the system is short (the number of active machines in the system is less than $M$), it is sent back as an operating machine, and the failure rate changes from $\alpha$ to $\lambda$. Otherwise, it acts as a standby machine with failure rate $\alpha$. Table 1 lists the notations used in this paper.

Such a system has potential applications in a network switch. A Digital Subscriber Line Access Multiplexer (DSLAM) is network switch equipment which collects multiple customer digital subscriber data and transfers it to a high-speed digital communications channel. We define two kinds of subscriber line cards: one is the $M$ primary line card and the other is the $W$ standby line card. If the primary line card fails, the DSLAM's redundancy feature allows the standby line card to successfully take over system operations with probability $1 - q$. The switchover from a standby line card to a primary line card may fail due to hardware or software issues with probability $q$. Both of the primary line cards and standby line cards can be considered to be repairable. The primary line cards fail independently of the state of the standby line cards and vice versa. Let the time between failures of the primary line cards and the time between failures of the standby line cards be exponentially distributed with parameters $\lambda$ and $\alpha$, respectively. Whenever a primary line card or a standby line card fails, it is immediately repaired in the order of breakdown with a time-to-repair which is generally distributed. Once a line card is repaired, its function is as good as new. Thus, this system can be modeled as an M/G/1 machine repairing system with standby switching failure.

**Table 1**
Notations used in this paper.

| Notation | Description |
|---|---|
| $M$ | Number of operating machines |
| $W$ | Number of standbys |
| $N$ | Number of machines in the system, where $N = M + W$ |
| $\lambda$ | Mean failure rate of an operating machine |
| $\alpha$ | Mean failure rate of a standby |
| $B(v)$ | Distribution function of the repair time |
| $b(v)$ | Probability density function of the repair time |
| $b$ | Mean repair time |
| $q$ | Probability of switching failure |
| $N(t)$ | Number of failed machines in the system at time $t$ |
| $V(t)$ | Remaining repair time for the failed machines under repair at time $t$ |
| $P_i(t)$ | Probability that there are $i$ failed machines in the system at time $t$, where $i = 0, 1, 2, \ldots, N$ |
| $s$ | Laplace transform variable |
| $\overline{P}_i(s)$ | Laplace-Stieltjes transform (LST) of $P_i(t)$ |
| $\overline{B}(s)$ | LST of the repair time |
| $L_S$ | Average number of failed machines in the system |
| $E[W]$ | Average number of standbys in the system |
| $E[I]$ | Average number of idle repairman in the system |
| $E[B]$ | Average number of busy repairman in the system |
| $S.R.$ | Average switching failure rate |
| $M.A.$ | Machine availability, the fraction of the total time that the machines are working |
| $O.U.$ | Operative utilization (the fraction of busy repairman) |
| $Av$ | System availability (the steady-state probability that at least $M$ machines are in operation) |

## 3. Steady-state results

### 3.1. Steady-state probability

We use the supplementary variable $V \equiv$ remaining repair time for the failed machines under repair. The state of the system at time $t$ is given by:

$N(t) \equiv$ number of failed machines in the system, and
$V(t) \equiv$ remaining repair time for the failed machines under repair.

Let us define:

$$P_i(v,t) = P\{N(t) = i, v < V(t) \leqslant v + dv\}, \quad v \geqslant 0, \quad i = 0, 1, 2, \ldots, N$$

$$P_i(t) = \int_0^{\infty} P_i(v,t)dv, \quad i = 0, 1, 2, \ldots, N$$

Using the state-transition-rate diagram shown in Fig. 1, we obtain the following differential equations

$$\frac{d}{dt}P_0(t) = -(M\lambda + W\alpha)P_0(t) + P_1(0,t) \tag{1}$$

$$\left(\frac{\partial}{\partial t} - \frac{\partial}{\partial v}\right)P_1(v,t) = -[M\lambda + (W-1)\alpha]P_1(v,t) + \lambda_0 P_0(v,t) + b(v)P_2(0,t), \tag{2}$$

$$\left(\frac{\partial}{\partial t} - \frac{\partial}{\partial v}\right)P_i(v,t) = -[M\lambda + (W-i)\alpha]P_i(v,t) + \lambda_{i-1}P_{i-1}(v,t) + b(v)P_{i+1}(0,t) + \sum_{k=0}^{i-2}\phi_{i-1-k}P_k(v,t), 2 \leqslant i \leqslant W, \tag{3}$$