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Optimal maintenance–replacement policy of products with competing failures after expiry of the warranty

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ABSTRACT

Usually, reliability of products with the warranty is maintained by manufactures during the warranty period. After expiry of the warranty, however, the product is actually a secondhand product from customers' perspective since it has operated for the warranty period. Through the review of literatures on maintenance after expiry of the warranty, there is no literature considering imperfect preventive maintenance at a time where the warranty expires. In this paper, we integrate imperfect preventive maintenance at a time where the warranty expires with age replacement, and propose a maintenance–replacement policy after expiry of the warranty for the product with two categories of competing failure modes. The proposed maintenance–replacement policy is that an imperfect preventive maintenance is performed first at a time where the warranty expires and then an age replacement is performed at an age. The performed imperfect preventive maintenance reduces failure rate function of maintainable failure modes by a random variable. Compared with traditional age replacement policy after expiry of the warranty, although the proposed maintenance–replacement policy incurs a preventive maintenance cost at the time where the warranty expires, it can improve greatly subsequent operation time as well as decrease greatly both operation cost and failure cost. We obtain the expected cost rate from perspective of the customer, under the proposed maintenance–replacement policy and renewable warranty policies. Conditions for existence and uniqueness of optimal solutions that minimize the expected cost rate are presented. Based on a numerical example, we demonstrate that the proposed maintenance–replacement policy is superior to traditional age replacement policy by comparing respectively the optimal preventive replacement age and the expected cost rate.

1. Introduction

To warrant a product for the consumer, the manufacturer provides usually warranty service which can keep reliability of the product. Warranty policies are usually classified into one-dimension warranty policy and two-dimension warranty policy. There are mainly two types of the one-dimension warranty policies. (1) Non-renewable warranty (NRW) policy, where the manufacturer presents repair or replacement service only during the original warranty period. (2) Renewable warranty (RW) policy, where the failed product is replaced by a new one at a cost to the manufacturer or at the prorated cost to the consumer and each replacement makes warranty renewable. Two types of the frequently used RW policies are renewable pro-rata replacement warranty (RPRRW) policy and renewable free replacement warranty (RFRW) policy. Their difference lies in that the former requests that manufacturer charge proportionally replacement cost to the consumer, while the latter doesn't request the manufacturer charge any replacement cost to the consumer. The most commonly used two-dimensional warranty policy is non-renewable two-dimensional warranty policy, where the manufacturer agrees to repair or replace the failed product free of charge before time limit or below usage limit, whichever occurs first. The basic information about warranty was provided by Blischke and Murthy (1992). Recently, a large quantity of literatures involving various issues associated with warranty have been published, such as Park and Pham (2010), Wu and Longhurst (2011), Varnosafaderani and Chukova (2012), Shahanaghi, Noorossana, Jalali-Naini, and Heydari (2013), Vahdani, Mahlooji, and Jahromi (2013), Park, Jung, and Park (2013), Tong, Liu, Men, and Cao (2014), Xie, Liao, and Zhu (2014), Li, Jia, Wang, and Zhao (2015), Liu, Wu, and Xie (2015), Wang, Liu, and Liu (2015), Yeh, Kurniati, and Chang (2015), Yeh et al. (2015), Huang, Gau, and Ho (2015),

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Nomenclature

RPRRW	renewable pro-rata replacement warranty
RFRW	renewable free replacement warranty
PM	preventive maintenance
PR	preventive replacement
p	probability of the type I failure when a failure occurs
q	probability of the type II failure when a failure occurs, where $q = 1 - p$
x	time, where $x \geqslant 0$
$F(x)$	distribution function of the maintainable failure modes
$\bar{F}(x)$	reliability function of the maintainable failure modes, where $\bar{F}(x) = 1 - F(x)$
$f(x)$	probability density function of the maintainable failure modes, where $f(x) = \frac{dF(x)}{dx}$
$\gamma(x)$	failure rate function of the maintainable failure modes, where $\gamma(x) = \frac{f(x)}{\bar{F}(x)}$
$G(x)$	distribution function of the non-maintainable failure modes
$\bar{G}(x)$	reliability function of the non-maintainable failure modes, where $\bar{G}(x) = 1 - G(x)$
$g(x)$	probability density function of the non-maintainable failure modes, where $g(x) = \frac{dG(x)}{dx}$
$\lambda(x)$	failure rate function of the non-maintainable failure modes
X_i	time between the $(i-1)^{th}$ and i^{th} occurrence of the type II failure, $i = 1, 2, \cdots$
$\bar{F}_q(x)$	reliability function of X_i, where $\bar{F}_q(x) = \exp(-q\int_0^x \gamma(u)du)$
$F_q(x)$	distribution function of X_i, where $F_q(x) = 1 - \bar{F}_q(x)$
w	warranty period
T	PR age
T^*	optimal value of T
υ	random variable, which represents that reduction in failure rate function of the maintainable failure modes due to imperfect PM, where $\upsilon \geqslant 0$
υ^*	optimal value of υ
$C_p(\upsilon)$	PM cost function with maintenance degree υ
$\Pr\{\bullet\}$	probability of the event $\bullet$
$F_w(x)$	distribution function of occurrence time of the first type II failure during the warranty period w
X_j	random variable, which represents replacement time of the jth product during the warranty period w
$E[X_j]$	the expected value of X_j
$\gamma_w(x)$	failure rate function of the maintainable failure modes of the product surviving the warranty period w, where $\gamma_w(x) = \gamma(w + x)$
$\lambda_w(x)$	failure rate function of the non-maintainable failure modes of the product surviving the warranty period w, where $\lambda_w(x) = \lambda(w + x)$
X_1^w	occurrence time of the first type II failure of the maintainable failure modes during the PR period T
$\bar{F}_{qw}(x)$	reliability function of X_1^w
$F_{qw}(x)$	distribution function of X_1^w, where $F_{qw}(x) = 1 - \bar{F}_{qw}(x)$
$f_{qw}(x)$	probability density function of X_1^w, where $f_{qw}(x) = \frac{dF_{qw}(x)}{dx}$
X_T	random variable, which is replacement time of the product during the PR period T, where $X_T = \min(X_1^w, T)$
$E[X_T]$	the expected value of X_T
$h(x)$	dependence function which represents the dependence between the maintainable failure modes and the non-maintainable failure modes when the imperfect PM is performed at time x
W	random variable, which is length of the actual warranty servicing period
$E[W]$	the expected value of W
L	random variable, which is life cycle length of the product
$E[L]$	the expected value of L
$\upsilon_1(w)$	random variable, which is the total failure cost during the warranty period w under the RPRRW policy
$\upsilon_2(w)$	random variable, which is the total failure cost during the warranty period w under the RFRW policy
$E[\upsilon_1(w)]$	the expected value of $\upsilon_1(w)$
$E[\upsilon_2(w)]$	the expected value of $\upsilon_2(w)$
M	the expected total failure cost during the warranty period w, where $M = E[\upsilon_1(w)]$ or $M = E[\upsilon_2(w)]$
$C(T)$	random variable, which is the total cost during the PR period T
$E[C(T)]$	the expected value of $C(T)$
$E[C(L)]$	the expected total cost during the life cycle
$C(T, \upsilon)$	the expected cost rate
c_0	cost coefficient of the cost function $C_p(\upsilon)$
c_r^{np}	unit unplanned replacement cost
c_r^p	unit planned replacement cost, where $c_r^p < c_r^{np}$
c_w	unit failure cost during the warranty period w
c_{mm}	unit failure cost during the PR period T
c_n	unit cost of a new product covered by the warranty
c_m	unit minimal repair cost during the PR period T

Lee, Cha, and Finkelstein (2016), Xie and Ye (2016), Yang, He, and He (2016) as well as Park and Pham (2016).

In recent years, with the increasing expected cost rate during the post-warranty period, maintenance policies after expiry of the warranty have attracted increasing attention. The post-warranty maintenance policy was researched early by Sahin and Polatoglu (1996), and it was subsequently studied by a great deal of literatures. See for example Jung, Lee, and Park (2000), Jung and Park (2003), Chien (2005, 2008, 2010), Jung, Han, and Park (2008), Jung, Park, and Park (2010), Park et al. (2013), Wu, Xie, and Ng (2011), Wu and Longhurst (2011), Bouguerra, Chelbi, and Rezg (2012), Shahanaghi et al. (2013), Tong et al. (2014), Xie et al. (2014), Jung, Park, and Park (2015) as well as Park and Pham (2016).

From a careful survey of literatures listed in the previous paragraph, it is inferred that the main focuses of these researches include the following aspects. Firstly, the following maintenance policies were used to maintain reliability of the product surviving the warranty period. (1) Age replacement policy, where the operating product is replaced at an age, such as Sahin and Polatoglu (1996), Chien (2010), Jung et al. (2015) as well as Park and Pham (2016). (2) Periodic preventive maintenance policy, where effect of the preventive maintenance is age reduction, such as Jung, Han, and Park (2008). (3) Block replacement policy, such as Park and Pham (2016). Secondly, these literatures were researched based on the following perspectives. (1) Keeping reliability of the product at the consumer's expense, such as Sahin and Polatoglu (1996), Jung et al. (2000), Jung and Park (2003), Chien (2005, 2008, 2010), Jung et al. (2008), Jung et al. (2010), Park et al. (2013) as well as Park and Pham (2016). (2) Buying the extended warranty period, such as Shahanaghi et al. (2013), Bouguerra et al. (2012), and Tong et al. (2014). (3) Using combination of the extended warranty and the self-maintenance, such as Wu and Longhurst (2011) as well as Jung et al. (2015). Thirdly, they