Computers & Industrial Engineering 98 (2016) 513–521

Contents lists available at ScienceDirect

Computers & Industrial Engineering

journal homepage: www.elsevier.com/locate/caie

Identification of congestion by means of integer-valued data envelopment analysis

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ARTICLE INFO

Article history:
Received 12 May 2014
Received in revised form 16 April 2016
Accepted 16 June 2016
Available online 22 June 2016

MSC:
90C05
90C90

Keywords:
Data envelopment analysis
Congestion
Mixed integer programming
Production possibility set

ABSTRACT

In traditional Data Envelopment Analysis (DEA) all inputs and outputs are assumed to take real values. However, this is not realistic in many practical situations and in some applications we may need to work with integer variables and parameters. Once we assume that the variables can take only integer values, we may need to review different concepts of DEA. In this paper we focus on congestion for integer-valued DEA. After introducing the preliminaries and axioms that we need to establish our models, we derive the associated production possibility set (PPS). This step is followed by introduction of a mixed integer programming (MIP) model to compute efficiency scores. More precisely, the solutions of the MIP model is used in evaluating the presence of congestion and in identifying the reasons. Finally, we apply our approach on a couple of empirical examples and report the results.

1. Introduction

As a branch of operations research, Data Envelopment Analysis (DEA) is a nonparametric approach in evaluating the efficiency of any Decision Making Unit (DMU) involved in a set of DMUs. After the pioneer work of Charnes, Cooper, and Rhodes (1978), DEA has been developed and studied in both theoretical and applied directions.

The conventional data envelopment analysis (DEA) models assume that each DMU has some input(s) and output(s). In the major DEA models, the DMUs are supposed to be independent. Furthermore, the classical DEA models suppose that the inputs and outputs of each DMU can take just real values. This assumption becomes unrealistic in some situations where some inputs and/or outputs can only take integer values. In order to overcome this inconvenience, several studies have been carried out with a focus on developing the fundamentals of Integer-Valued DEA and, consequently, introducing suitable Integer-Valued DEA models. Lozano and Villa (2006, 2007) presented the pioneering works in this direction. They have studied the differences between real-valued and integer-valued DEA models. In their study, Lozano and Villa introduced a Mixed Integer Linear Programming (MILP) model that insures the integrality constraints of the computed solutions. In the same context of integer-valued DEA, the works of Kazemi Matin and Kuosmanen (2009) and Kuosmanen and Kazemi Matin (2009) are based on defining some integrality axioms. The axiomatic approach of Kazemi Matin and Kuosmanen (2009) and Kuosmanen and Kazemi Matin (2009) consists in developing Production Possibility Sets (PPS) that are used in proposing a variety of MILP models. The objective of these models consists in measuring the efficiency of DMUs. The main difference of traditional DEA model with above integer-valued DEA models is in the reference point. In fact, reference point in integer-valued DEA models is integer. The studies on integer-valued DEA have been followed by several works that are based on different models and concepts of DEA. For example, Du, Chen, Chen, Cook, and Zhu (2012) developed models for computing the additive integer-valued efficiency and the super-efficiency scores. Based on SBM DEA model, Chen, Du, Huo, and Zhu (2012) proposed an integer DEA model in order to

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http://dx.doi.org/10.1016/j.cie.2016.06.017
0360-8352/

compute the efficiency scores in presence of undesirable integer data. Kazemi Matin and Emrouznejad (2011) proposed a mixed integer programming model for efficiency evaluation of integer valued DMUs under the constraints of bounded outputs. In a recent work, Wu et al. introduced a mixed-objective integer DEA model in which, the objective consists in maximizing the input excesses as well as output shortfalls, simultaneously.

In the same context of previous studies on integer-valued DEA, we address an important concept in DEA, but under the condition of integrality. More precisely, we are interested in examining *congestion* in the context of integer-valued DEA. More precisely, we extend the previous works of Wei and Yan (2004) to integer-valued framework and we propose an effective approach for evaluation of congestion in DEA under integrality constraints.

From econometric point of view, congestion is an economic state that is resulted from excessive investments in one or some inputs; in other words, by reducing the values of one or some inputs, we can get more outputs, without increasing (respectively, decreasing) other inputs (respectively, other outputs). This concept has received particular attentions after the works of Fare and Svensson (1980), Fare and Grosskopf (1983) and Fare, Grosskopf, and Lovell (1985), since then other researches started working on congestion and developing models and approaches to identify this kind of inefficiency (Brockett, Cooper, Ding, Golden, & Ruee, 2004; Cooper, Thompson, & Thrall, 1996). The pioneer works of Fare and Svensson (1980), Fare and Grosskopf (1983) and Fare et al. (1985) have been resulted in introducing a DEA (input-oriented) model to detect the presence of congestion. However, this model is not able to detect congestion in some situations (Cooper, Seiford, & Zhu, 2004). In the same context, Cooper et al. (1996) introduced an alternative DEA approach for studying congestion. In comparison to the model of Fare et al. (1985), the model of Cooper et al. (1996) is output-oriented and contains some additional convexity constraints (Cooper, Seiford, & Kaoru, 2000a). The investigations of Cooper et al. (1996) have been extended by Brockett et al. (2004) and Brockett, Cooper, Shin, and Wang (1998) that is known under the name of *BCSW* approach. The studies of Wei and Yan (2004) are based on identifying the necessary and sufficient conditions that certify the presence of congestion under different forms of returns to scale (i.e., constant, increasing, and decreasing). Tone and Sahoo (2004) suggested a DEA framework to investigate the influence of congestion on scale elasticity in production.

But if the inputs and outputs would be limited to take integer values, In this case how can we identify congestion and also decrease the amount in inputs and increase in outputs to eliminate congestion? In addition, reference point after eliminating congestion must be integrated. In this article, the MIP models will be presented to assess integer congestion, which give the integer reference point to eliminate congestion.

In order to present our contributions, the current paper is organized as follows. In Section 2, the notations that we are going to use are presented and we give a technical introduction to integer-valued DEA. The main contributions of the current paper are presented in Section 3. This section is devoted to models and methods that we propose in order to identify congestion under integrality constraints. A couple of empirical and illustrative examples are presented in Section 4. The final section contains some concluding remarks and perspectives.

2. Integer-valued data envelopment analysis

Let us consider n decision making units (DMUs) denoted by $\{(x_j, y_j) : j = 1, \ldots, n\}$. We assume that each DMU produces the same set of outputs by consuming the same set of inputs and the sole difference may be in the quantity of inputs and outputs. For each DMU_j (where $j = 1, \ldots, n$), we denote the non-negative input and output vectors by $x_j = (x_{1j}, \ldots, x_{mj})^t$ and $y_j = (y_{1j}, \ldots, y_{sj})^t$ (where, t is the sign of transposition). For the sake of simplicity in the notations, we use $X = [x_1, \ldots, x_n]^t$ and $Y = [y_1, \ldots, y_n]^t$ to denote, respectively, the $m \times n$ input matrix and the $s \times n$ output matrix.

Conventional DEA models assume that all data are allowed to take positive real values. However, in many practical cases, some inputs and/or outputs can only take integer values. Lozano and Villa (2006, 2007) were the pioneers in paying attention to this difference. They introduced integer constraints into DEA models and proposed a mixed integer linear programming (MILP) model for evaluating the efficiency of DMUs. In the same context, Kazemi Matin and Kuosmanen (2009) did some studies in integer-valued DEA. The works of Kazemi Matin and Kuosmanen (2009) are based on some axioms. For the sake of completeness of the current paper, we present a summarized description of their axioms. To this aim, we suppose that T is the production possibility set (PPS) of the integer-valued DEA, defined by

$$T = \{(x, y) | x \in Z_+^m \text{ can produce } y \in Z_+^s\}.$$

The axioms of Kazemi Matin and Kuosmanen (2009) are as follows:

(1) Envelopment: $(x_j, y_j) \in T : \forall j = 1, \ldots, n$
(2) Natural disposability: $(x, y) \in T$ and $(u, v) \in Z_+^{(m+s)} : y \geqslant v \Rightarrow (x + u, y - v) \in T$
(3) Natural convexity: if $(x_1, y_1), (x_2, y_2) \in T$ and $(x, y) = \lambda(x_1, y_1) + (1 - \lambda)(x_2, y_2)$ where $\lambda \in [0, 1]$, then $(x, y) \in Z_+^{(m+s)} \Rightarrow (x, y) \in T$.
(4) Natural divisibility: $(x, y) \in T$ and $\exists \lambda \in [0, 1] : (\lambda x, \lambda y) \in Z_+^{(m+s)} \Rightarrow (\lambda x, \lambda y) \in T$
(5) Natural augmentability: $(x, y) \in T$ and $\exists \lambda \geqslant 1 : (\lambda x, \lambda y) \in Z_+^{(m+s)} \Rightarrow (\lambda x, \lambda y) \in T$

These axioms are integer variants of the standard axioms in conventional DEA. More precisely, the axioms differ from standard axioms only in the type of input and output vectors that must be integer-valued. More detailed description of these axioms can be found in Kazemi Matin and Kuosmanen (2009) and Kuosmanen and Kazemi Matin (2009).

According to the presented axioms, one can construct a variety of PPSs (see Kazemi Matin & Kuosmanen, 2009). In this paper, we will use the following PPS satisfying the Axioms 1, 2, and 3:

$$T_{VRS}^{IDEA} = \left\{ (x, y) \in Z_+^{(m+s)} | x \geqslant \sum_{j=1}^{n} \lambda_j X_j; y \leqslant \sum_{j=1}^{n} \lambda_j Y_j; \sum_{j=1}^{n} \lambda_j = 1, \lambda_j \geqslant 0, \forall j \right\}.$$

Based on this PPS, Kazemi Matin and Kuosmanen (2009) and Kuosmanen and Kazemi Matin (2009) presented an input-oriented radial model. In their model, the input and output variables are classified in two categories. The classification is based on the type of the variables, i.e., whether they are continuous or integer. In the following, we present an output-oriented version of their model. In this model, I stands for *integer input/output* and the subsets of integer-valued and real-valued inputs, integer-valued and real-valued outputs are denoted by I^I, I^{NI}, O^I and O^{NI}, respectively (where ^{NI} means *Non-Integer*).