Computers & Industrial Engineering 98 (2016) 531–542

Contents lists available at ScienceDirect

Computers & Industrial Engineering

journal homepage: www.elsevier.com/locate/caie

The symmetrical interval intuitionistic uncertain linguistic operators and their application to decision making

Fanyong Meng [a,*], Xiaohong Chen [a,b,1]

[a] Business School, Central South University, Changsha 410083, China
[b] School of Accounting, Hunan University of Commerce, Changsha 410205, China

ARTICLE INFO

Article history:
Received 18 October 2013
Received in revised form 25 June 2015
Accepted 30 October 2015
Available online 23 November 2015

Keywords:
Multi-attribute decision making
Interval intuitionistic uncertain linguistic set
Shapley function
2-Additive measure

ABSTRACT

Interval intuitionistic uncertain linguistic sets are an important generalization of fuzzy sets, which well cope with the experts' qualitative preferences as well as reflect the interval membership and non-membership degrees of the uncertain linguistic term. This paper first points out the issues of the operational laws on interval intuitionistic uncertain linguistic numbers in the literature, and then defines some alternative ones. To consider the relationship between interval intuitionistic uncertain linguistic sets, the expectation and accuracy functions are defined. To study the application of interval intuitionistic uncertain linguistic sets, two symmetrical interval intuitionistic uncertain linguistic hybrid aggregation operators are defined. Meanwhile, models for the optimal weight vectors are established, by which the optimal weighting vector can be obtained. As a series of development, an approach to multi-attribute decision making under interval intuitionistic uncertain linguistic environment is developed, and the associated example is provided to demonstrate the effectivity and practicality of the procedure.

1. Introduction

Linguistic variables are an effective tool to deal with the experts' qualitative preference. It well deals with the situations where the problem is too complex or too ill-defined to use quantitative expressions. For example, when evaluating the "comfort" or "design" of a car, terms like "good", "medium", and "bad" (Levrat, Voisin, Bombardier, & Bremont, 1997) can be used. The linguistic approach is an approximate technique, which represents qualitative aspects by means of linguistic variables (Zadeh, 1975), namely, the variable values are words or sentences in a linguistic term set rather than numbers. Since Zadeh (1975) first introduced the concept of linguistic variables, it has been used in many fields such as information retrieval (Fontana, 2001; Herrera-Viedma & Peis, 2003; Herrera-Viedma & Lopez-Herrera, 2007, 2010; Herrera-Viedma, Lopez-Herrera, Luque, & Porcel, 2007), investment risk assessment (Fenton & Wang, 2006; Liu, Zhang, & Liu, 2011; Shevchenko, Ustinovichius, & Andruševičius, 2008), and decision making (Dong, Xu, & Yu, 2009; Herrera & Herrera-Viedma, 2000a; Liu, 2009; Martínez, Ruan, & Herrera, 2010; Wei, 2011; Wei, Zhao, & Lin, 2013; Xu, 2004a, 2004b, 2007a; Zhou & Chen, 2013). On the other hand, some extending forms are developed such as 2-tuple linguistic variables (Herrera & Martínez, 2000; Martínez & Herrera, 2012; Wei, 2010; Xu & Wang, 2011), uncertainty linguistic variables (Park, Gwak, & Kwun, 2011; Xu, 2004c, 2006a, 2006b; Xu & Wu, 2013), and hesitant fuzzy linguistic term sets (Rodríguez, Martínez, & Herrera, 2013).

Recently, some researchers (Wang & Li, 2009) noticed that linguistic variables only reflect the experts' qualitative preferences, and do not consider the membership and non-membership degrees of an element to a particular concept. To better express the experts' qualitative preferences, using linguistic values (Levrat et al., 1997) and Atanassov's intuitionistic fuzzy sets (AIFSs) (Atanassov, 1986), Wang and Li (2009) proposed the concept of intuitionistic linguistic sets, which do not only give the experts' qualitative preferences but also consider the membership and non-membership degrees of their qualitative preferences. Later, Liu and Jin (2012) further defined the concept of intuitionistic uncertain linguistic sets, whilst Liu (2013) presented interval intuitionistic uncertain linguistic sets (IIULSs), which further facilitates effectively representing inherent imprecision and uncertainty in the human decision making process. For simplicity, the author further introduced interval intuitionistic uncertain linguistic numbers (IIULNs) and defined some operations on IIULNs in a similar

* Corresponding author at: Yuelu District, Changsha Hunan Province 410083, China. Tel.: +86 18254298903.

E-mail addresses: mengfanyongtjie@163.com (F. Meng), cxh@csu.edu.cn (X. Chen).

1 Address: Yuelu District, Changsha Hunan Province 410083, China.

http://dx.doi.org/10.1016/j.cie.2015.10.020
0360-8352/

way to De, Biswas, and Roy (2000) and Xu (2007b). Although the IIULNs give us more information of the considered problems, there exist some undesirable properties for the operations on IIULNs as Beliakov, Bustince, Goswami, Mukherjee, and Pal (2011) pointed for the operations on AIFSs.

The purpose of this paper is to develop an approach to multi-attribute decision making under interval intuitionistic uncertain linguistic environment with incomplete weight information and interactive characteristics. To do this, this paper first defines several alternative operations on IIULNs, which can be seen as a natural extension of the operational laws on uncertainty linguistic variables. The given operations avoid the need for complex and explicit constructions (Liu, 2013). Then, based on the given operations on IIULNs, two symmetrical hybrid aggregation operators are defined to calculate the comprehensive attribute values of alternatives. Furthermore, some models are established, by which the optimal weighting vectors can be obtained. To research the application of the defined operations, an approach to interval intuitionistic uncertain linguistic multi-attribute decision making is developed.

The rest of this paper is organized as follows: In Section 2, some concepts relate to IIULNs are briefly reviewed, and the existing issues in the literature (Liu & Jin, 2012; Liu, 2013) are pointed. In Section 3, several alternative operations on IIULNs are defined that eliminate the mentioned issues. Furthermore, two hybrid aggregation operators on IIULNs are presented. In Section 4, models for the optimal weight vectors are built, which can deal with the situations where the weight information is not exactly known. In Section 5, a new method to multi-attribute decision making under interval intuitionistic uncertain linguistic environment is developed, and an example is selected to illustrate the effectivity and practicality of the proposed procedure. The conclusion is made in the last section.

2. Basic concepts of IIULNs

To deal with the qualitative fuzzy preferences, the experts usually use linguistic variables rather than numerical ones. The linguistic approach is an approximate technique, which represents qualitative aspects by means of linguistic variables. Let $S = \{s_i | i = 1, 2, \ldots, t\}$ be a linguistic term set with odd cardinality. Any label, s_i represents a possible value for a linguistic variable, and it should satisfy the following characteristics (Herrera & Herrera-Viedma, 2000b):

(1) The set is ordered: $s_i > s_j$, if $i > j$.
(2) Max operator: max $(s_i, s_j) = s_i$, if $s_i \geqslant s_j$.
(3) Min operator: min $(s_i, s_j) = s_i$, if $s_i \leqslant s_j$.

For example, to assess the responsiveness of an engine, one may use the following linguistic term set: $S = \{s_1$: very poor, s_2: poor, s_3: indifferent, s_4: good, s_5: very good$\}$.

Later, Xu (2004c) extended the discrete linguistic term set S to a continuous linguistic term set $\bar{S} = \{s_\alpha | s_1 \leqslant s_\alpha \leqslant s_t, \alpha \in [1, t]\}$. To further facilitate representing inherent uncertainty of the human decision making process, Xu (2004c) gave the concept of uncertain linguistic variables as follows: an uncertain linguistic variable can be denoted by $\tilde{s} = [s_\alpha, s_\beta]$, where $s_\alpha, s_\beta \in \bar{S}$ with s_α and s_β being the lower and upper limits. Some operational laws on uncertain linguistic variables can be seen in the literature (Xu, 2004c, 2006b).

Although the (uncertain) linguistic variables can well cope with the experts' qualitative fuzzy preferences, they cannot reflect the membership and non-membership degrees of an element to a concrete concept. Based on uncertain linguistic variables (Xu, 2004c) and Atanassov's interval-valued intuitionistic fuzzy sets (AIVIFSs) (Atanassov & Gargov, 1989), Liu (2013) defined the IIULSs, which can be seen as an extension of intuitionistic linguistic sets (Wang & Li, 2009) and intuitionistic uncertain linguistic sets (Liu & Jin, 2012).

Definition 1. (Liu, 2013). An interval intuitionistic uncertain linguistic set (IIULS) A in $X = \{x_1, x_2, \ldots, x_n\}$ is expressed by

$$A = \left\{ \left\langle x_i | \left([s_{\theta(x_i)}, s_{\tau(x_i)}], [u_l(x_i), u_u(x_i)], [v_l(x_i), v_u(x_i)] \right) \right\rangle | x_i \in X \right\},$$

where $[u_l(x_i), u_u(x_i)]$ and $[v_l(x_i), v_u(x_i)]$ respectively represent the interval membership and non-membership degrees of the element $x_i \in X$ to the uncertain linguistic variable $[s_{\theta(x_i)}, s_{\tau(x_i)}]$ with $[u_l(x_i), u_u(x_i)] \subseteq [0, 1], [v_l(x_i), v_u(x_i)] \subseteq [0, 1]$ and $u_u(x_i) + v_u(x_i) \leqslant 1$ for each $x_i \in X$.

For brevity, Liu (2013) further gave the definition of IIULNs. An interval intuitionistic uncertain linguistic number (IIULN) $\tilde{\alpha}$ is defined by $\tilde{\alpha} = ([s_{\theta(\alpha)}, s_{\tau(\alpha)}], [u_l(\alpha), u_u(\alpha)], [v_l(\alpha), v_u(\alpha)])$, where $[u_l(\alpha), u_u(\alpha)]$ and $[v_l(\alpha), v_u(\alpha)]$ respectively represent the interval membership and non-membership degrees of the uncertain linguistic variable $[s_{\theta(\alpha)}, s_{\tau(\alpha)}]$ with $[u_l(\alpha), u_u(\alpha)] \subseteq [0, 1], [v_l(\alpha), v_u(\alpha)] \subseteq [0, 1]$ and $u_u(\alpha) + v_u(\alpha) \leqslant 1$.

Similar to the operations on uncertain linguistic variables (Xu, 2004c, 2006b) and on AIVIFSs (Xu, 2007b), Liu (2013) defined the following operations on IIULNs.

Let $\tilde{\alpha} = ([s_{\theta(\alpha)}, s_{\tau(\alpha)}], [u_l(\alpha), u_u(\alpha)], [v_l(\alpha), v_u(\alpha)])$ and $\tilde{\beta} = ([s_{\theta(\beta)}, s_{\tau(\beta)}], [u_l(\beta), u_u(\beta)], [v_l(\beta), v_u(\beta)])$ be any two IIULNs, then some operations between $\tilde{\alpha}$ and $\tilde{\beta}$ are defined by

(i) $\tilde{\alpha} \oplus \tilde{\beta} = ([s_{\theta(\alpha)+\theta(\beta)}, s_{\tau(\alpha)+\tau(\beta)}], [1 - (1 - u_l(\alpha))(1 - u_l(\beta)), 1 - (1 - u_u(\alpha))(1 - u_u(\beta))], [v_l(\alpha) v_l(\beta), v_u(\alpha) v_u(\beta)])$;
(ii) $\tilde{\alpha} \otimes \tilde{\beta} = ([s_{\theta(\alpha)\theta(\beta)}, s_{\tau(\alpha)\tau(\beta)}], [u_l(\alpha) u_l(\beta), u_u(\alpha) u_u(\beta)], [1 - (1 - v_l(\alpha))(1 - v_l(\beta)), 1 - (1 - v_u(\alpha))(1 - v_u(\beta))])$;
(iii) $\lambda\tilde{\alpha} = \left([s_{\lambda\theta(\alpha)}, s_{\lambda\tau(\alpha)}], [1 - (1 - u_l(\alpha))^\lambda, 1 - (1 - u_u(\alpha))^\lambda], [v_l(\alpha)^\lambda, v_u(\alpha)^\lambda]\right) \lambda \in [0, 1]$;
(iv) $\tilde{\alpha}^\lambda = \left([s_{\theta(\alpha)^\lambda}, s_{\tau(\alpha)^\lambda}], [u_l(\alpha)^\lambda, u_u(\alpha)^\lambda], [1 - (1 - v_l(\alpha))^\lambda, 1 - (1 - v_u(\alpha))^\lambda]\right) \lambda \in [0, 1]$.

Considering the application of IIULNs in decision making, Liu (2013) gave an order relationship between IIULNs in a similar way to that on AIVIFSs (Xu & Chen, 2007).

Let $\tilde{\alpha} = ([s_{\theta(\alpha)}, s_{\tau(\alpha)}], [u_l(\alpha), u_u(\alpha)], [v_l(\alpha), v_u(\alpha)])$ be an IIULN, Liu (2013) defined the expected function $E(\tilde{\alpha})$ of $\tilde{\alpha}$ by $E(\tilde{\alpha}) = s_{\frac{(\theta(\alpha)+\tau(\alpha))(u_l(\alpha)+u_u(\alpha)+2-v_l(\alpha)-v_u(\alpha))}{8}}$ and presented the accuracy function $H(\tilde{\alpha}) = s_{\frac{(\theta(\alpha)+\tau(\alpha))(u_l(\alpha)+u_u(\alpha)+v_l(\alpha)+v_u(\alpha))}{4}}$ to evaluate the accuracy degree of $\tilde{\alpha}$. Then, the following order relationship (Liu, 2013), for any two IIULNs $\tilde{\alpha}$ and $\tilde{\beta}$, is defined by

If $E(\tilde{\alpha}) < E(\tilde{\beta})$, then $\tilde{\alpha} \prec \tilde{\beta}$.

If $E(\tilde{\alpha}) = E(\tilde{\beta})$, then $\begin{cases} H(\tilde{\alpha}) = H(\tilde{\beta}) \Rightarrow \tilde{\alpha} = \tilde{\beta} \\ H(\tilde{\alpha}) < H(\tilde{\beta}) \Rightarrow \tilde{\alpha} \prec \tilde{\beta} \end{cases}$.

Based on the given operational laws and the order relationship, Liu (2013) defined some aggregation operators and researched the application of IIULNs in decision making. However, there are some undesirable properties of the given operational laws as well as the given order relationship.

Similar to the issue pointed by Beliakov et al. (2011) for the operations on AIFSs, the operations (iii) and (iv) defined on IIULNs (Liu, 2013) do not preserve the order relationship given above under multiplication or exponentiation by a scalar: $\tilde{\alpha} \prec \tilde{\beta}$ cannot guarantee $\lambda\tilde{\alpha} \prec \lambda\tilde{\beta}$ or $\tilde{\alpha}^\lambda \prec \tilde{\beta}^\lambda$ for $\lambda \in [0, 1]$. Let us consider Examples 1 and 2 below.