Computers & Industrial Engineering 93 (2016) 28–35

Contents lists available at ScienceDirect

Computers & Industrial Engineering

journal homepage: www.elsevier.com/locate/caie

A bi-level formulation for DEA-based centralized resource allocation under efficiency constraints

S. Hakim, A. Seifi *, A. Ghaemi

Dept. of Industrial Engineering and Management Systems, Amirkabir University of Technology, 424 Hafiz St., P.O. Box. 15875-4413, Tehran, Iran

ARTICLE INFO

Article history:
Received 19 January 2015
Received in revised form 30 August 2015
Accepted 24 December 2015
Available online 31 December 2015

Keywords:
Data envelopment analysis
Resource allocation
Bi-level optimization
Efficiency
Effectiveness

ABSTRACT

This paper presents a bi-level data envelopment analysis (DEA) model for centralized resource allocation considering lower bounds on efficiencies of decision making units (DMUs). The DMUs are controlled by a central unit which has the authority to allocate limited resources to them so that overall organization effectiveness is maximized. The upper-level model is concerned with determining input resources and output targets as key decision variables while imposing lower bounds on BCC efficiencies of all DMUs evaluated in the lower-level model. The model is optimized for organizational effectiveness (total outputs minus total inputs) while trying to improve the efficiencies of all DMUs. We prove that such a bi-level DEA model can be converted to a single level optimization problem. Numerical results are presented and compared with various examples taken from the literature.

1. Introduction

An important application of data envelopment analysis (DEA) involves allocation of limited resources of a large organization to its subordinate decision making units (DMUs). The central unit has the responsibility to determine input resources and output targets for each DMU while considering the efficiencies of DMUs simultaneously. Most resource allocation models cited in the literature deal with either total outputs or total inputs without considering DMUs' efficiencies (see Lozano & Villa, 2004). Such mechanisms ignore the decision making power of the subordinates and could lead to disagreement of DMUs with organizational goal settings. The purpose of this paper is to develop a resource allocation model based on DEA approach to optimize the overall organization effectiveness (total output minus total inputs) while allowing each DMU to best estimate its own efficiency.

Golany, Phillips, and Rousseau (1993) suggested a five-step approach to allocate resources across all DMUs simultaneously. The objective function of their model is claimed to be weighted according to DMU efficiencies. However, their efficiency measurement based on an additive DEA model is not a standard efficiency measurement. Furthermore, they do not determine output targets.

Golany and Tamir (1995) proposed a model based on output-oriented DEA model for resource allocation whose main objective is to maximize the sum of outputs of all DMUs. A DEA model is used to characterize an efficient frontier and the production possibility set based on some observed input and output values. Their model also includes some constraints that reflect limitation of resource availabilities and some bounds on allocated resources.

Färe, Grabowski, Grosskopf, and Kraft (1997) presented a generalized output-oriented model and compared its results with those derived from a standard output oriented DEA model while allowing for reallocation of fixed amounts of resources. Such a comparison shows how the desired efficiency can be potentially achieved. However, this approach is applicable if each of the inputs is only used for the production of a specific output.

Athanassopoulos (1995) proposed a goal programming model based on DEA by setting targets for inputs and outputs of each DMU and global inputs and outputs. It minimizes the deviations from some predetermined targets as the objective function. He later changed his original model and presented another goal programming model based on the multiplier form (Athanassopoulos, 1998).

Beasley (2003) presented a DEA based model that is more general than the standard DEA approach presented in Charnes, Cooper, and Rhodes (1978). This non-linear model is used for resource allocation with the objective of maximizing the average efficiency of all DMUs. The model has two drawbacks. The first is that it does not consider the organizational effectiveness. The second drawback is that only simple bounds on input resources and output targets of each DMU is considered and other constraints needed for defining feasible production plans were neglected.

* Corresponding author. Tel.: +98 21 64545377.
E-mail address: aseifi@aut.ac.ir (A. Seifi).

http://dx.doi.org/10.1016/j.cie.2015.12.020
0360-8352/

Korhonen and Syrjänen (2004) developed a formal interactive approach based on DEA. Various assumptions are made in their model. One of their assumptions is that the efficiency of each unit stays constant in the next period. Another assumption made is to allow proportional scaling of the existing production plan. The model has multiple objectives of maximizing the total output target of each type produced by all DMUs. The observed input and output values are used to estimate a production possibility set while assuming that re-allocation does not change this set. However, this assumption is not valid and the production possibility set will change based on new set of inputs and outputs.

Lozano and Villa (2004) and Lozano, Villa, and Adenso-Diaz (2004) presented centralized resource allocation models. The first cited model tries to minimize total input resources while the objective of the second model is to maximize total output targets. However, they do not directly consider efficiency in their models. Since they do not put any bounds on input resources and output targets, they are able to map all the units on the efficient frontier and thereby achieve full efficiencies for all units in light of what is proved in Du, Liang, Chen, and Bi (2010). In practice, it is necessary to consider such bounds as a reflection of admissible changes in inputs or outputs as considered in Beasley (2003). Asmild, Paradi, and Pastor (2009) considered the centralized model proposed by Lozano and Villa (2004) and modified it to adjust only inefficient units while keeping the inputs and outputs of efficient units unchanged. This policy is questionable since the inputs and outputs of efficient units could also be changed for the sake of organizational effectiveness. Lozano and Villa (2005) modified their original centralized model (proposed in 2004) to consider the possibility of downsizing the number of existing DMUs.

Du et al. (2010) developed DEA-based models to determine new input and output plans for all individual units in the next production period while considering the output targets to be less than some forecasted demands. Vaz, Camanho, and Guimarães (2010) extended the model in Färe et al. (1997) and used it for a retail store. The main distinctions are that their model determines overall outputs by allocating many inputs as compared with the determination of efficiencies and allocation a single input in Färe et al. (1997) model.

Wu (2010) proposed a bi-level formulation that is used for allocating resources for each DMU independently. Each DMU is assumed to be composed of two decentralized subsystems: leader and follower. Some outputs of the leader (intermediate outputs) are considered as inputs to the follower. The upper-level problem determines resources allocation for the leader while the lower-level problem does the same for the follower. The objective function is to minimize the total cost of inputs using a predetermined cost vector for each input. Since the intermediate outputs are considered as inputs to DMUs, those outputs are minimized. The situation modeled by Wu (2010) does not match with the needs of practical application and is far from the concepts used in all other DEA based resource allocation models cited in the literature. No comparison was made in that paper with other DEA based resource allocation models in the literature.

Guedes, Milioni, Avrllar, and Silva (2012) and Milioni, Avellar, Gomes, and Mello (2011) presented parametric DEA models for allocating only one input. The main assumption of Guedes et al. (2012) is the spherical shape of the efficiency frontier. Milioni, Avellar, Gomes et al. (2011) assumed the ellipsoidal shape for the efficient frontier. Milioni, Avellar, Rabello, and Freitas (2011) proposed parametric DEA model for distributing only one output. They assumed that efficiency frontier have hyperbolic shape. Mar-Molinero, Prior, Segovia, and Portillo (2012) simplified the centralized resource allocation model proposed by Lozano and Villa (2004) and applied their model to Spanish public schools. The number of variables in their simplified model grows linearly with the number of DMUs as opposed to quadratic growth in Lozano and Villa's (2004) model. All of the above-mentioned parametric DEA models may not be suitable for a general data set because of assuming some predefined shape for the efficient frontier. Moreover, those models assumed constant returns to scale (CRS) which is again not a practical assumption. They were able to project all DMUs right on the efficient frontier improving the efficiency of all DMUs in their objective function while neglecting bounds on input resources and output targets.

Hossenzadeh Lotfi, Hatami-Marbini, Agrell, Aghayi, and Gholami (2013) and Hatami-Marbini, Tavana, Agrell, Hossenzadeh Lotfi, and Ghelej Beigi (2015) presented a resources allocation model and used a common set of weights to obtain DMUs efficiencies. Hatami-Marbini et al. (2015) used a scaling scheme to reduce both the inputs and outputs equally whereas the model proposed by Hossenzadeh Lotfi et al. (2013) found 100% efficiency for all DMUs after resource allocation. Fang (2015) presented a stepwise model for resource allocation by which all DMUs become efficient at the last step.

Du, Cook, Liang, and Zhu (2014) proposed a DEA-based model for allocating some fixed cost and resources. The authors claimed that their model would maintain or improve the efficiencies achieved in the previous period. However, their results showed that some of DMUs' efficiencies were reduced. The reason for obtaining lower efficiency for one of DMU is that their model tries to maximize the firm incremental output and could not choose correct weights for computing the efficiencies as is commonly done in DEA models. We show in this paper how such deficiency can be avoided using a bi-level DEA model.

In this paper, we present a bi-level DEA model for centralized resource allocation considering lower bounds on efficiencies of DMUs. The DMUs are assumed to be controlled by a central unit which has the authority to allocate limited resources among them so that the overall organizational effectiveness is maximized. The upper-level model is concerned with determining input resources and output targets while the efficiencies of DMUs are evaluated in the lower-level model. Furthermore, using DEA concepts, it is possible to find solutions in the feasible production plans. We prove that such a bi-level DEA model can be converted to a single level optimization problem. The contribution of this paper is mainly concentrated on the proper bi-level formulation of centralized resource allocation based on DEA concepts.

The rest of this paper is organized as follows. Section 2 presents the proposed bi-level formulation of DEA-based resource allocation model. Section 3 discusses the solution method. In Section 4, various numerical examples from the literature are solved using the proposed model and the results are compared with those of other resource allocation models. Section 5 gives some concluding remarks.

2. A bi-level formulation of DEA-based model for resource allocation

In this section, we present a bi-level DEA**-based** model for centralized resource allocation under efficiency constraints. The upper-level model is concerned with determining input resources and output targets while the efficiencies of DMUs are evaluated in the lower-level model.

2.1. Notation and definitions

The following notations are used throughout this paper:

n: is the number of DMUs;
m: is the number of input measures;