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A multi-period output DEA model with consistent time lag effects

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ABSTRACT

Conventional data envelopment analysis (DEA) models assume that the inputs of a specific period are consumed to produce the outputs of the same period. However, in some applications, the inputs of a period can be thought to partially contribute to the outputs of several subsequent periods. This can be described as time lag effects for performance evaluation. A few researches have presented DEA models using the time lag weights of input or output factors to address the time lag effects. In those models, the weights for the time lag effects can vary by period. In this paper, we propose another DEA model with consistent weights for time lag effects throughout the periods. The proposed model is more realistic and more discriminative than the existing time lag model. We present the results of a case example for comparison of the proposed model and the existing time lag model.

1. Introduction

Data envelopment analysis (DEA), developed by Charnes, Cooper, and Rhodes (1978), is a method to evaluate the performance of decision making units (DMUs) by measuring the relative efficiency of a DMU with the inputs and outputs of a specific period. There are many extensions of the original CCR model in view of various points. In particular, Banker, Charnes, and Cooper (1984) suggested the BCC model to extend the CCR model to accommodate technologies that exhibit variable returns to scale. Some alternative DEA approaches have been reported to use various types of data, for example, use of categorical data (Banker & Morey, 1986), qualitative data (Cook, Kress, & Seiford, 1996), and ordinal data (Toloo & Nalchigar, 2011). Kim, Park, and Park (1999) developed an imprecise DEA model to deal with imprecise data. Uncertainty or fuzziness in data set was also discussed in the context of DEA by Wu (2009) and Dotoli, Epicoco, Falagario, and Sciancalepore (2015). Amin and Toloo (2007) suggested an integrated DEA model to find the most efficient DMU. Toloo and Nalchigar (2011) extended to identify the most efficient supplier in presence of both cardinal and ordinal data.

The fundamental goal of DEA models is not to rank but to group DMUs into two sets, that is, efficient and inefficient DMUs. However, decision-makers are often interested in a complete ranking in the evaluation of the DMUs beyond the dichotomized classification. This is the reason why the lack of discrimination has been discussed frequently in the DEA literature. This is also the reason for the growing interest in complete ranking techniques in the context of DEA even though there are tremendous papers about multiple-criteria decision making (MCDM) (Yoon & Hwang, 1995; Zavadskas, Turskis, & Kildiene, 2012) and analytic hierarchy process (AHP) (Saaty, 1980). There are various kinds of approaches to improve discrimination power of classical DEA models. Some researchers considered the weight restrictions for the improvement. Allen, Athanassopoulos, Dyson, and Thanassoulis (1997) classified the types of weight restriction in detail as cone ratio weight restriction (Charnes, Cooper, Wei, & Huang, 1989), specification of relative bounds on input or output weights (Wong & Beasley, 1990), and absolute weight bounds (Podinovski & Athanassopoulos, 1998; Roll, Cook, & Golany, 1991). Adler, Friedman, and Sinuany-Stern (2002) reviewed and classified various ranking methods which is another approach for the improvement. Andersen and Petersen (1993) suggested a super-efficiency model to discriminate and rank efficient DMUs. Charnes, Clark, Cooper, and Golany (1985) proposed the idea to rank efficient DMUs by the number of times they appear in the reference sets of inefficient units. Doyle and Green (1994) used a cross evaluation matrix, which consists of efficiency and cross-efficiency values, for ranking DMUs. Jeong and Ok (2013) modified the cross evaluation matrix by replacing efficiency with super-efficiency to get a full ranking of DMUs.

Some researchers proposed integrated models with analytic hierarchy process. Wang, Liu, and Elhag (2008) suggested an integrated methodology of AHP and DEA to evaluate bridge risks of bridge structures, based on which the maintenance priorities of

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the bridge structures can be decided. Yang and Kuo (2003) also proposed an integrated approach of AHP and DEA to solve a plant layout design problem. Wei, Zhang, and Zhang (2000) developed an inverse DEA model for estimating inputs or outputs. The inverse DEA model can estimate the amount of outputs (inputs) should be changed to keep the determined efficient score when a DMU changes some of its inputs (outputs). Hadi-Vencheha and Foroughib (2006) also proposed a generalized DEA model for the estimation problem.

As above mentioned, the standard DEA model has been extended in various viewpoints. However, these extended DEA models were developed under the basic assumption that inputs of a specific period are consumed to produce outputs in the same period. This underlying assumption may not be valid in some situations such as performance evaluation on R&D activity, marketing activity or educational activity. In those situations, outputs of a specific period can be thought to be produced by consumption of not only inputs in the same period but also the inputs in multiple previous periods. In other words, inputs of a specific period can be considered to contribute to the outputs of several subsequent periods as well as the same period. For example, publication of academic papers and application of patents are important outputs of research activity. But the papers or patents may be the results from research efforts over several years. In other words, there are some time lag between input period and output period. Furthermore, the length of time lag is uncertain and depends on areas in which performance is to be evaluated. Thus, general DEA models cannot be applied to obtain efficiency scores in presence of time lag effect.

To deal with such situations, a couple of researchers have proposed DEA models considering time lag effects in recent years. Özpeynici and Köksalan (2007) developed the multi-period input (MPI) model, which is based on an integrated model developed by Post and Spronk (1999), to capture the time lag between the inputs and outputs in DEA. The model is formulated under the assumption that the outputs of a DMU in a specific period are the results of the inputs of several previous periods. They also gave a MPI model to restrict input weights by adding weight range constraints like the approach to specify relative bounds of weight (Wong & Beasley, 1990). But, those constraints are not to keep the time lag weight consistently period by period. Zhang and Jeong (2012a, 2012b) introduced a multi-period output (MPO) model to capture the time lag effect in another viewpoint. That is, this model was developed in the perspective that the inputs of a specific period contribute to the outputs of several subsequent periods. In those two models, the magnitude of time lag effects is represented by time lag weights and they are determined freely to maximize the efficiency score of DMUs. As a result, the time lag weights for the same output or input factors are varies from period to period. In other words, the magnitude of time lag effects of an input (output) factor can be changed according to the spent (yielded) period. This is not realistic because the characteristics of lag effect in productions of outputs depends on not input period but the time difference between input period and output period. Thus, it is more realistic that the magnitude of time lag effects is consistent over periods for the same input or output factors. In this paper, We suggest a modified model adopting the consistency of the time lag weights. The model is a multi-period output model that has the same time lag weight throughout the measurement periods for each output factor.

In the next section, we compare two schemes to consider the time lag effects and present the existing DEA models based on these schemes. In Section 3, we propose a modified DEA model considering consistent time lag effects and present some properties between the efficiency scores by the modified model and the existing time lag model. In Section 4, we provide a case example of a Korean R&D program and compare the results of the proposed model and the existing time lag model. Finally, we present some concluding remarks.

2. Time lag effects and DEA models

In this section, we introduce the methods to adopt the time lag effects and give two existing DEA models adopting the time lag effect.

2.1. Schemes for considering time lag effects

Time lag refers to an interval of time between two related phenomena. In measuring the efficiency of DMUs with inputs and outputs, the time lag effect indicates the time it might take to transform inputs into outputs, and that the inputs can contribute to the outputs of multiple subsequent periods.

Two types of models adopting time lag effects were proposed from different perspectives. The first one considered the time lag effect from the perspective of the outputs in a period. In the model, the outputs in period t are the results of the inputs of the previous D periods, $t, t-1, \ldots, t-D+1$. That is, D is the length of the time periods to handle the time lag effects in the model. We call this scheme the MPI model because the inputs of multiple periods contribute to the outputs of a single period. Conversely, the second one considered that the inputs in period t contribute to the outputs of subsequent D periods, $t, t+1, \ldots, t+D-1$. This second model is called the MPO model because the inputs of a single period contribute to the outputs of multiple subsequent periods. Fig. 1 shows two schemes for considering the time lag effects when $D = 3$. As shown in the figure, the MPI model considers the outputs of period 3 to be the results of the inputs of periods 1–3, whereas the MPO model considers the inputs of period 1 to contribute to the outputs of periods 1–3.

According to the types considering time lag effect, the efficiency score of a period should be defined in different form. In MPI model, the efficiency score of a period can be defined as the ratio of the outputs of a single period to the inputs of multiple periods. On the other hand, it can be defined as the ratio of outputs of multiple periods to the inputs of a single period in MPO model. For example, the MPI model defines the efficiency of period 3 as the ratio of the outputs of period 3 to the inputs of periods 1–3, while the MPO model defines the efficiency of period 1 as the ratio of the outputs of periods 1–3 to the inputs of period 1. Then, two models have a non-linear objective function but it can be linearized by fixing the weighted sum of inputs to 1.

2.2. Existing multi-period DEA models for time lag effects

In this subsection, we present the existing models considering the time lag effects explained in 2.1. Let's consider the efficiencies of n DMUs during T time periods under the time lag effects. We assume that all DMUs use m different inputs to produce s different outputs in each period. Let X_{ijt} and Y_{rjt} denote the ith ($i = 1, \ldots, m$) input and rth ($r = 1, \ldots, s$) output of DMU j at period t, respectively.

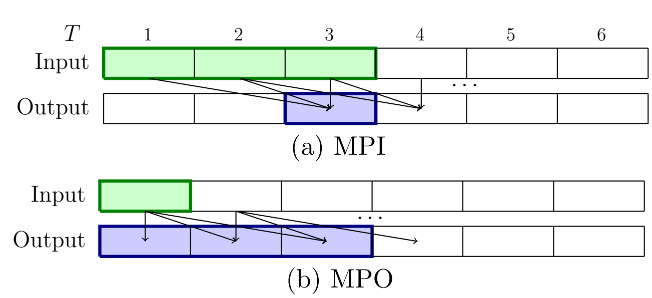

Fig. 1. Two schemes for time lag effects.